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扎列伊状态

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The spacetime discreteness of causal set theory has enabled the formulation of novel spacetime dynamics. In these so-called ”growth” dynamics, a causal set spacetime is generated probabilistically by means of a random walk on certain tree structures. The first growth dynamics - the classical sequential growth models - were proposed more than two decades ago, and their study has furthered our understanding of general covariance and covariant observables within causal set theory. In this setting, labels take the place of spacetime coordinates so that general covariance takes the form of label invariance and covariant observables are those order-theoretic properties of the causal set which are label-independent. In recent years, these insights have led to a new formulation of growth dynamics which makes no reference to labels. Here, we present an overview of these (manifestly) covariant growth dynamics.

因果集理论的时空离散性为新型时空动力学的构造提供了基础。在这类所谓的“增长”动力学中，因果集时空通过特定树结构上的随机游走概率生成。首个增长动力学——经典序贯增长模型——于二十多年前被提出，对该模型的研究深化了我们对因果集理论中广义协变与协变可观测量的理解。在该框架下，标签取代了时空坐标，因此广义协变表现为标签不变性，而协变可观测量正是因果集中不依赖于标签的序理论性质。近年来，这些研究成果催生了不依赖标签的增长动力学新表述。本文将对这些 (显式) 协变增长动力学做综述。

Keywords

关键词

General covariance - Spacetime dynamics - Observables

广义协变 - 时空动力学 - 可观测量

Introduction

引言

In causal set theory - where the continuum spacetime of general relativity is replaced by a discrete causal set - spacetime points and their coordinates are replaced by causal set elements and their ”labels.” Thus, Einstein’s struggle between the formulation of generally covariant laws of nature and the intrinsic (in)distinguishability of spacetime points [1-3] manifests in causal set theory as tension between label dependence and label independence. This tension can be expressed via a myriad of interrelated questions: What is the physical status which one should assign to the causal set elements and to their labels? Should we conflate the ”label” of an element with the ”intrinsic identity” of an element or should they be considered separately? What are the precise mathematical concepts which are best suited for formulating a physical theory of causal sets? In particular, could the theory be formulated without any reference to ”labels”?

在因果集理论中，广义相对论的连续时空被离散因果集取代，时空点及其坐标被替换为因果集元素及其「标号」。因此，爱因斯坦在广义协变自然定律的表述与时空点内在 (不) 可区分性之间的挣扎 [1-3]，在因果集理论中体现为标号依赖与标号无关性之间的张力。这种张力可以通过一系列相互关联的问题表达: 我们应当赋予因果集元素及其标号何种物理地位？我们应当将元素的「标号」与元素的「内在同一性」等同，还是将二者区分开？哪些精确的数学概念最适合构建因果集物理理论？具体而言，该理论能否完全不借助「标号」来表述?

The importance of understanding general covariance in any given theory is heightened within the path integral approach to quantum dynamics [4-6] whose most appealing feature in the context of gravity is arguably its compatibility with general covariance: the integral sums over complete spacetime histories and therefore does not require a foliation or a distinguished time parameter, while covariant ”observables” can be defined independently of observers as attributes of histories.

在量子动力学的路径积分方法 [4-6] 中，理解广义协变性在任意理论中的重要性被进一步凸显。在引力语境下，路径积分最吸引人的特点无疑是它与广义协变性的兼容性: 路径积分对所有完整的时空历史求和，因此不需要叶状分解或特殊的时间参数，同时协变「可观测量」可以作为历史的属性，独立于观测者定义。

Defining the path integral for quantum gravity remains a challenge [7, 8] , and one may be justified in regarding the path integral as a guiding principle rather than an exact prescription. In causal set theory, the path integral is replaced by a discrete ”sum-over-histories.” The challenges in its definition and interpretation may be summarized by three open questions:

定义量子引力的路径积分仍是一个挑战 [7, 8]，因此将路径积分视为指导原则而非精确的方案是合理的。在因果集理论中，路径积分被离散的「历史求和」取代。其定义与解释层面的挑战可以归纳为三个开放性问题:

What is the domain of the sum-over-histories?

历史求和的定义域是什么?

What is the amplitude by which each history should be weighted?

每个历史应当被赋予多大的振幅权重?

What are the physical observables?

物理可观测量是什么?

These are the questions that the growth dynamics program aims to answer [9- 29]. A growth dynamics is a probabilistic process in which a causal set comes into being ex nihilo by accretion of elements. This growth process plays a dual role: it embodies the sum-over-histories (e.g., by providing a mechanism from which the action is emergent), and it offers a novel route for accounting for the passage of time within physics [19-22, 30]. Crucially, the growth does not happen in time - it constitutes the passage of time. The birth of an element is the happening of that event, while the existence of an element signifies that the event has already happened. Thus heuristically, the growth process is a physical process whose phenomenological manifestation is the passage of time. (While work in the field is usually motivated by this interpretation, its results are not contingent on it.)

这些就是增长动力学方案旨在回答的问题 [9-29]。增长动力学是一种概率过程: 因果集从无到有，通过元素的不断累积产生。这个增长过程扮演着双重角色: 它体现了历史求和 (例如，提供了作用量从中涌现的机制)，还为在物理学中解释时间流逝提供了全新路径 [19-22, 30]。至关重要的是，增长并非发生在时间之中——它本身就构成了时间流逝。一个元素诞生就是一个事件的发生，而元素的存在则标志着该事件已经发生。因此从直观上看，增长过程是一种物理过程，其现象学表现就是时间流逝。(尽管该领域的研究通常受这一解释驱动，研究结论并不依赖于它。)

The archetype of growth dynamics for causal sets are the classical sequential growth (CSG) models of [9]. In these models, the causal set elements are born one after another (in a sequence, hence the name sequential) and form relations with each other according to model-dependent probabilities. The functional form of the probabilities satisfies mathematical constraints motivated by local causality and general covariance. The latter, known as discrete general covariance, states that the probability that the first n born elements form some causal set $\widetilde{C}_n$ is equal to the probability that they form any causal set which is order-isomorphic to $\widetilde{C}_n$.

因果集增长动力学的原型是文献 [9] 提出的经典序贯增长 (CSG) 模型。在这些模型中，因果集元素依次诞生 (按顺序排列，因此得名「序贯」)，并根据模型依赖的概率彼此建立因果关系。概率的函数形式满足由局部因果性和广义协变性导出的数学约束。后者被称为离散广义协变性，其内容是: 最先 n 诞生的元素构成某一因果集 $\widetilde{C}_n$ 的概率，等于它们构成任意一个与 $\widetilde{C}_n$ 序同构的因果集的概率。

In the language of growth dynamics, the domain of the sum-over-histories is the sample space of the growth process (i.e., the causal sets or histories which can be grown by the process). The role of the amplitude by which each history is weighted is played by the probabilities which govern the stochastic growth of the causal set. And the observables are sets of histories (known in the probability literature as events). Thus, the open questions which are posed by the sum-over-histories take on a more concrete form:

用增长动力学的语言来说，历史求和的定义域就是增长过程的样本空间 (即该过程所能生长出的因果集或历史)。支配每个历史权重的振幅角色，由控制因果集随机增长的概率承担，而可观测量则是历史的集合 (在概率论文献中称为事件)。因此，历史求和提出的开放性问题转化为更具体的形式:

Should the growth process produce ”labeled” or ”unlabeled” causal sets? Should it produce all infinite causal sets or only those which are past-finite?

增长过程应当产生「带标号」还是「不带标号」的因果集？应当产生所有无限因果集，还是仅产生过去有限的因果集？

How should the probabilities be constrained to obtain physical dynamics? And how can the probabilities be generalized into complex amplitudes so that the resulting dynamics exhibits quantum interference?

应当如何约束概率才能得到物理动力学？又如何将概率推广为复振幅，使最终动力学展现出量子干涉？

Which events or observables have a physical interpretation? Which of these should we interpret as ”local” and which as ”global”?

哪些事件或可观测量具有物理解释？其中哪些应当被解释为「局域」的，哪些是「全局」的？

In this chapter we take our cue from the first of these questions, reviewing the state-of-the-art growth dynamics for ”unlabeled” causal sets. We call these dynamics manifestly covariant dynamics or simply covariant dynamics for short. But our discussion will touch upon all the above themes, revealing the interconnectedness of domain, amplitude, and observables.

本章我们围绕上述第一个问题展开，综述针对「无标号」因果集的最新增长动力学研究进展。我们将这类动力学称为显协变动力学，简称协变动力学。但我们的讨论将涉及上述所有主题，揭示定义域、振幅与可观测量之间的相互关联。

From the start, one can identify three challenges. First, unlabeled causal sets are mathematically more difficult to handle (e.g., enumerating unlabeled graphs is generally more difficult than enumerating labeled graphs [31]), echoing the difficulties often encountered in physics when working with global degrees of freedom. Second, a common approach to obtaining a physical dynamics is to impose invariance under certain gauge transformations. Indeed, the CSG models were obtained by imposing the discrete general covariance condition which is akin to the requirement that the Einstein-Hilbert action is invariant under diffeomorphisms. Therefore even if a label-independent framework did exist at the level of the kinematics, how are the physically meaningful dynamics to be picked out from the plethora of available models? Finally, our intuitive notion of growth is inherently sequential: elements are born one after the other in a kind of global time which renders the elements distinguishable (e.g., in a CSG model, each element is labeled by its position in the sequence of births). How does one reconcile a physical process of becoming with manifest covariance? The prevailing view in causal set theory is that one should seek a form of ”asynchronous becoming,” namely, a growth process in which elements are born in a partial (not a total) order [19, 21, 22] . What could it mean for elements to be born in a partial order? It is the role of mathematics to make sense of notions which lie beyond our everyday experience, and it may be that new mathematics is what is needed to better understand asynchronous becoming and its consequences for the nature of time. It has been suggested in [20] that this could be achieved via a ”novel and exotic” framework in which questions such as ”which element is born at stage n ?” are left unanswered (not because of ignorance, but because they are unphysical). As we shall see, this notion is affirmed by our current understanding of covariant dynamics [32-34].

从一开始就能确定三个挑战。第一，无标签因果集在数学上更难处理 (例如，枚举无标签图通常比枚举有标签图更难 [31])，这和物理学中处理全局自由度时常遇到的困难一致。第二，获得物理动力学的常用方法是要求其在特定规范变换下保持不变。实际上，CSG 模型正是通过施加离散广义协变性条件得到的，这一条件类似于爱因斯坦-希尔伯特作用量在微分同胚下不变的要求。因此，哪怕在运动学层面已经存在不依赖标签的框架，又该如何从大量可用模型中筛选出具有物理意义的动力学？第三，我们对增长的直观理解本质上是时序性的: 元素在某种全局时间中依次生成，这使得元素可区分 (例如在 CSG 模型中，每个元素由它在生成序列中的位置标记)。我们该如何调和生成的物理过程与显式协变性？因果集理论的主流观点认为，应当寻找一种“异步生成”形式，即元素按偏序 (而非全序) 生成的增长过程 [19, 21, 22] 。元素按偏序生成意味着什么？数学的作用本就是赋予超出日常经验的概念意义，或许要更好理解异步生成及其对时间本质的影响，恰恰需要新的数学。文献 [20] 提出，这可以通过一个“新颖特殊”的框架实现，该框架不对“哪个元素在阶段 n 生成”这类问题作答 (并非因为我们无知，而是这类问题本身是非物理的)。正如我们将会看到，当前对协变动力学的认知也印证了这一观点 [32-34]。

Labels and Label Invariance

标签与标签不变性

Should we conceive of the elements of a causal set as distinguishable or indistinguishable? Mathematically, the elements of a causal set are distinguishable (in so far as a causal set is a set), and the notion of labeling these elements appears in pure mathematics and in its applications, including in causal set theory where labels naturally arise within the CSG models. But the clear distinguishable-indistinguishable divide is blurred by opposing physical notions within causal set theory. On the one hand, the correspondence between spacetime volume and the number of spacetime elements forces one ”to accept that the elements of the causal set are real, and that volume measurements ’count’ them in much the same way that weighing a copper ingot ’counts’ the number of atoms it comprises” [35]. On the other hand, it is a postulate of causal set theory that no information is contained in any individual identity or label of the elements, so that stripped of ordering the elements are physically indistinguishable. Is this indistinguishability compatible with the elements’ physical existence? Can you measure the cardinality of an antichain? These subtle questions underpin the discussion of covariance and labels within causal set theory. Largely, the answers given by the community to these last two questions have been a yes. Considered separately, each element has no distinguishing characteristic, but arranged together in a partial order they give rise to a meaningful structure whose properties include, for example, a notion of cardinality. These considerations, as well as the analogy between coordinates in the continuum and labels in the discrete, have led to the understanding of general covariance as label invariance within causal set theory [9,12,13]. This understanding is our starting point for the discussion of covariant dynamics.

我们应当将因果集的元素设想为可区分还是不可区分的？从数学角度看，因果集的元素是可区分的(因为因果集本身就是集合)，给这些元素赋予标记的概念既出现在纯数学中，也出现在其各类应用里，包括因果集理论中——标记自然存在于 CSG 模型中。但在因果集理论内部，对立的物理想念模糊了可区分与不可区分之间的清晰界限。一方面，时空体积与时空元素数量之间的对应关系迫使人们“接受因果集元素是真实存在的，体积测量对它们的‘计数’，本质上和称量铜锭得到它所含原子数的‘计数’是同一回事”[35]。另一方面，因果集理论有一个公设: 元素的任何个体同一性或标记都不包含信息，因此去掉序关系后，元素在物理上是不可区分的。这种不可区分性和元素的物理存在相容吗？你能测量反链的基数吗？这些微妙问题构成了因果集理论中协变性与标记讨论的基础。学界对最后这两个问题的答案基本都是肯定的。单独来看，每个元素都没有区分特征，但当它们共同排布在一个偏序中时，就会形成有意义的结构，该结构的性质就包含基数概念等。这些考量，加上连续体坐标和离散标记之间的类比，使得因果集理论中将广义协变性理解为标记不变性 [9,12,13]。这一理解就是我们讨论协变动力学的出发点。

The Notion of Labeling

标号的概念

A partial order is a pair, $(\Pi, \prec)$, where $\prec$ is a transitive, irreflexive relation on the ground-set Π . A linear order (also known as a total order) is a partial order in which any two elements are comparable. A labeling of a set Π is a mapping λ from Π to an index set $\mathcal{I}$. When Π carries additional structure, it may be desirable that the labeling reflect this additional structure. In particular, when labeling a partially ordered set $(\Pi, \prec)$, one often endows the index set with a total order $\ll$ and requires that the labeling is order-preserving, i.e., $x \prec y \Rightarrow \lambda(x) \ll \lambda(y) \forall x, y \in \Pi$. Thus, a labeling of $(\Pi, \prec)$ arranges the elements of Π into a linear order $\ll$ which is compatible with the partial order $\prec$. This notion of labeling is shared by the various definitions that can be found in the literature [36-38].

偏序是一个序对 $(\Pi, \prec)$ ，其中 $\prec$ 是基集 Π 上的传递非自反关系。线性序 (也称为全序) 是任意两个元素都可比较的偏序。集合 Π 的一个标号是从 Π 到索引集 $\mathcal{I}$ 的映射 λ。当 Π 具有附加结构时，通常要求标号能够反映该附加结构。具体来说，对标量偏序集 $(\Pi, \prec)$ 进行标号时，我们通常会为索引集赋予一个全序 $\ll$ ，并要求标号是保序的，即满足 $x \prec y \Rightarrow \lambda(x) \ll \lambda(y) \forall x, y \in \Pi$ 。因此，$(\Pi, \prec)$ 的标号将 Π 的元素排列为与偏序 $\prec$ 相容的线性序 $\ll$ 。文献 [36-38] 中的各类定义都采用了这一标号概念。

An important special case is the natural labeling where the index set of labels is (a subset of) the natural numbers. In words, a natural labeling is an enumeration of the causal set elements which respects the partial order. Thus while the labels contain an element of gauge, they reflect the partial order through their compatibility with it. We may draw an analogy with a familiar example from the continuum: inertial coordinates on Minkowski spacetime. Inertial coordinates provide a labeling of spacetime points, where each spacetime point p is labeled by a coordinate vector $\left(t_p, \bar{x}_p\right)$. The coordinates reflect the causal structure through the time coordinate, since if a spacetime point p is in the causal past of another spacetime point q , then $t_p < t_q$.

自然标号是一个重要的特例，它的标签索引集是自然数 (的子集)。简言之，自然标号是尊重偏序关系的因果集元素枚举。因此，尽管标号包含规范元素，它们通过与偏序的相容性来反映偏序。我们可以举一个连续统中大家熟悉的例子类比: 闵可夫斯基时空上的惯性坐标系。惯性坐标系给时空点做标号，每个时空点 p 被标上一个坐标向量 $(t_p, \bar{x}_p)$ 。坐标通过时间坐标反映因果结构: 如果时空点 p 位于另一个时空点 q 的因果过去，那么 $t_p < t_q$ 。

Equipped with the notion of labeling, one may use the term labeled partial order to mean a partial order together with a natural labeling of it [13]. In practice, one often discusses labeled partial orders without specifying the ground-set Π by repackaging the information contained in the order relation $\prec$ and the natural labeling into a partial order on a set of natural numbers [13, 26, 39] .

有了标号的概念之后，我们可以用“标号偏序”指代带有自然标号的偏序 [13]。在实践中，讨论标号偏序时通常不指定基集 Π ，而是将序关系 $\prec$ 和自然标号中包含的信息重新打包为自然数集合 [13, 26, 39] 上的偏序。

The term unlabeled partial order is borrowed from graph theory and means that the elements of the partial order are indistinguishable when stripped of the order relations. Therefore, an unlabeled partial order is not in fact a partial order but an order-isomorphism equivalence class of partial orders. Which partial orders are contained in a given equivalence class depends on one's universe of discourse (e.g., partial orders on a specified ground-set).

“无标号偏序”这一术语借自图论，其含义是: 去掉序关系后，偏序的元素是不可区分的。因此，无标号偏序本质上不是一个偏序，而是偏序的序同构等价类。一个给定等价类包含哪些偏序取决于讨论的论域 (例如，指定基集上的偏序)。

Terminology

术语

A causal set (or causet) is a locally finite partial order. For any natural number $n < \infty$, let $[0, n]$ denote the set $\{0, 1, \dots, n\}$ (devoid of any ordering).

因果集 (简称 causet) 是局部有限的偏序。对任意自然数 $n < \infty$ ， 令 $[0, n]$ 表示集合 $\{0, 1, \dots, n\}$ (不带有任何序结构)。

Definition 1 (Labeled causet). A labeled causet is any causet $([0, n], \prec)$ or $(\mathbb{N}, \prec)$ satisfying $x \prec y \Rightarrow x < y$.

定义 1(标记因果集)。标记因果集是任意满足 $x \prec y \Rightarrow x < y$ 的因果集 $([0, n], \prec)$ 或 $(\mathbb{N}, \prec)$ 。

Definition 2 (n -causet). An n -causet is a labeled causet of cardinality n .

定义 2(n -因果集)。n -因果集是基数为 n 的标记因果集。

Our universe of discourse contains all labeled causets and their subcausets. (Note that a subcauset of a labeled causet is not necessarily a labeled causet because its ground-set may not be an interval of integers of the form $\{0, 1, \ldots, n\}$.) Given some $n > 0$, we denote the set of n -causets by $\widetilde{\Omega}(n)$. The set of finite labeled causets is denoted by $\widetilde{\Omega}(\mathbb{N})$, i.e., $\widetilde{\Omega}(\mathbb{N}) := \bigcup_{n>0} \widetilde{\Omega}(n)$. The set of infinite labeled causets is denoted by $\widetilde{\Omega}$. Labeled causets and their subcausets are denoted by capital Roman letters with a tilde, e.g., $\widetilde{C}$. We often (but not always) use a subscript to denote the cardinality of an n -causet, e.g., $\widetilde{C}_n$.

我们的论域包含所有标记因果集及其子因果集。(注意: 标记因果集的子因果集不一定是标记因果集, 因为它的基集不一定是形如 $\{0, 1, \ldots, n\}$ 的整数区间。) 给定任意 $n > 0$ ，我们将 n -因果集的集合记为 $\widetilde{\Omega}(n)$ 。有限标记因果集的集合记为 $\widetilde{\Omega}(\mathbb{N})$ ，即 $\widetilde{\Omega}(\mathbb{N}) := \bigcup_{n>0} \widetilde{\Omega}(n)$ 。无限标记因果集的集合记为 $\widetilde{\Omega}$ 。标记因果集及其子因果集用带波浪号的大写罗马字母表示，例如 $\widetilde{C}$ 。我们通常 (但不总是) 用下标表示 n -因果集的基数，例如 $\widetilde{C}_n$ 。

Definition 3 (Order). An order (or ”unlabeled causet”) is an order-isomorphism equivalence class of labeled causets.

定义 3(序)。序 (或称“无标记因果集”) 是标记因果集的序同构等价类。

We denote orders by capital Roman letters without a tilde. Given an order, the Hasse diagrams of its representatives differ from each other only by the labeling of nodes (i.e., they are graph-isomorphic). Therefore, we represent an order by a Hasse diagram without node labels (Fig. 1).

我们用不带波浪号的大写罗马字母表示序。对给定的序，其代表元的哈塞图仅在节点标记上有区别 (即它们是图同构的)。因此我们用不带节点标记的哈塞图表示一个序 (图 1)。

Definition 4 (Cardinality of an order). The cardinality of an order is defined to be the cardinality of a representative of it.

定义 4(序的基数)。序的基数定义为其任意代表元的基数。

We denote the cardinality of an order C by $|C|$.

我们将序 C 的基数记为 $|C|$ 。

Definition 5 (n -order). An n -order is an order of cardinality n .

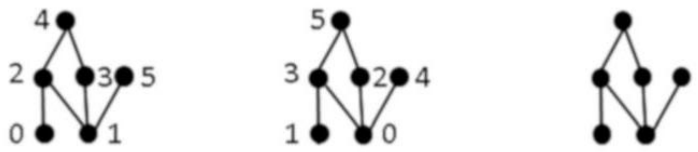
定义 5(n -序)。n -序是基数为 n 的序。

In other words, an n -order is an order whose representatives are n -causets. We often (but not always) use a subscript to specify the cardinality of an n -order, e.g., C_n .

换句话说，n -序是其代表元均为 n -因果集的序。我们通常 (但不总是) 用下标指定 n -序的基数，例如 C_n 。

Fig. 1 $\widetilde{C}$ and $\widetilde{C}'$ are order-isomorphic labeled causets. Each is a representative of the order C , shown on the right as a Hasse diagram without labels

图 1 $\widetilde{C}$ 和 $\widetilde{C}'$ 是序同构的标记因果集，二者都是序 C 的代表元，该序右侧以不带标记的哈塞图展示



$\widetilde{C}$ $\widetilde{C}'$ C

We use $\Omega(n), \Omega(\mathbb{N})$, and Ω to denote the set of n-orders, the set of finite orders, and the set of infinite orders, respectively. Note that these are equivalent to the quotient spaces $\widetilde{\Omega}(n)/\cong, \widetilde{\Omega}(\mathbb{N})/\cong$, and $\widetilde{\Omega}/\cong$, where $\cong$ denotes equivalence under order-isomorphism.

我们分别用 $\Omega(n), \Omega(\mathbb{N})$ 和 Ω 表示 n-序的集合、有限序的集合和无限序的集合。注意这些都等价于商空间 $\widetilde{\Omega}(n)/\cong, \widetilde{\Omega}(\mathbb{N})/\cong$ 和 $\widetilde{\Omega}/\cong$，其中 $\cong$ 表示序同构下的等价关系。

Similar to the way an order "inherits" the cardinality of its representatives, an order is future-finite if its representatives are future-finite, etc. We may also refer to an element of an order, meaning an element of a representative of it - the meaning should be clear from the context.

与序“继承”其代表元基数的方式类似，如果序的代表元是未来有限的，那么该序就是未来有限的，依此类推。我们也会提及序的元素，指的是该序某个代表元的元素——具体含义可根据上下文明确区分。

Definition 6 (Stem - labeled). A stem in a labeled causet $\widetilde{C}$ is a finite subcauset $\widetilde{D} \subseteq \widetilde{C}$ which contains its own past, i.e., if $x \in \widetilde{D}$ and $y \prec x$ in $\widetilde{C}$, then $y \in \widetilde{D}$.

定义 6(标记茎): 标记因果集 $\widetilde{C}$ 中的茎是包含自身过去的有限子因果集 $\widetilde{D} \subseteq \widetilde{C}$，即若 $x \in \widetilde{D}$ 且 $y \prec x$ 在 $\widetilde{C}$ 中，则 $y \in \widetilde{D}$。

In particular, given any labeled causet $\widetilde{C}$ and an integer n satisfying $0 \leq n \leq |\widetilde{C}|$, the restriction of $\widetilde{C}$ to the interval $[0, n]$ is a stem in $\widetilde{C}$.

特别地，对任意满足 $0 \leq n \leq |\widetilde{C}|$ 的标记因果集 $\widetilde{C}$ 和整数 n，将 $\widetilde{C}$ 限制在区间 $[0, n]$ 上得到的就是 $\widetilde{C}$ 中的一个茎。

Definition 7 (Stem - unlabeled). A finite order S is a stem in the order C if there exists a representative of S which is a stem in some representative of C. When S is a stem in C, we may also say that S is a stem in any representative $\widetilde{C}$ of C.

定义 7(非标记茎): 若有限序 S 存在一个代表元，是序 C 某个代表元中的茎，则称有限序 S 是序 C 中的茎。当 S 是 C 中的茎时，我们也可以称 S 是 C 任意代表元 $\widetilde{C}$ 中的茎。

Hence, the meaning of "stem" depends on the context (Fig. 2).

因此，“茎”的含义取决于上下文 (图 2)。

Definition 8 (n-stem). An n -stem is a stem of cardinality n .

定义 8(n-茎): n -茎是基数为 n 的茎。

The notion of rogue is closely related to that of stem.

反常因果集的概念与茎的概念密切相关。

Definition 9 (Rogue - labeled). An infinite labeled causet $\widetilde{C} \in \widetilde{\Omega}$ is a rogue if there exists some $\widetilde{D} \in \widetilde{\Omega}$ such that $\widetilde{C} \ncong \widetilde{D}$ and $S \in \Omega(n)$ is a stem in $\widetilde{D}$ if and only if S is a stem in $\widetilde{C}$. We say that $\widetilde{C}$ and $\widetilde{D}$ are equivalent rogues or a rogue pair.

定义 9(标记反常因果集): 若无穷标记因果集 $\widetilde{C} \in \widetilde{\Omega}$ 存在某个 $\widetilde{D} \in \widetilde{\Omega}$，满足 $\widetilde{C} \ncong \widetilde{D}$，且 $S \in \Omega(n)$ 是 $\widetilde{D}$ 中的茎当且仅当 S 是 $\widetilde{C}$ 中的茎，则称 $\widetilde{C} \in \widetilde{\Omega}$ 为反常因果集。我们称 $\widetilde{C}$ 和 $\widetilde{D}$ 为等价反常因果集，或反常对。

Definition 10 (Rogue - unlabeled). An order is a rogue if its representatives are rogues. Equivalently, C and D are a rogue pair when $S \in \Omega(n)$ is a stem in D if and only if S is a stem in C .

定义 10(非标记反常因果集): 若一个序的所有代表元都是反常因果集，则该序是反常因果集。等价地，当且仅当 $S \in \Omega(n)$ 是 D 中的茎当且仅当 S 是 C 中的茎时，C 和 D 是一个反常对。

Rogue equivalence, denoted by $C{\sim_R}D$, is an equivalence relation on Ω . An example is shown in Fig. 3.

反常等价记作 $C{\sim_R}D$，是定义在 Ω 上的等价关系。图 3 给出了一个示例。

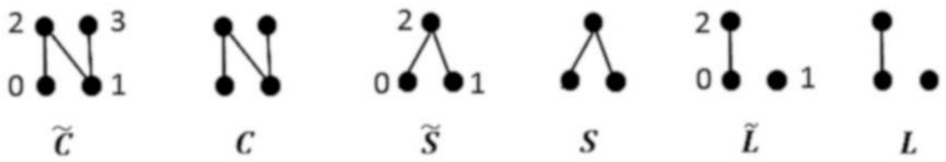


Fig. 2 Labeled causets $\widetilde{C}, \tilde{S}$, and $\tilde{L}$ are representatives of orders C, S , and L , respectively. $\tilde{S}$ is a stem in $\widetilde{C}.\tilde{L}$ is not a subcauset of $\widetilde{C}$ so it is not a stem in $\widetilde{C}.S$ and L are stems in $\widetilde{C}$ and in C

图 2 标记因果集 $\widetilde{C}, \tilde{S}$ 和 $\tilde{L}$ 分别是序 C, S 和 L 的代表元。$\tilde{S}$ 是 $\widetilde{C}.\tilde{L}$ 中的茎，它不是 $\widetilde{C}$ 的子因果集，因此它不是 $\widetilde{C}.S$ 中的茎，且 L 是 $\widetilde{C}$ 和 C 中的茎

Fig. 3 C is a countable union of 2-chains and D is the union of C with a single unrelated element. C and D have the same stems - any union of finitely many 2-chains and a finite, unrelated antichain - hence, C and D are equivalent rogues

图 3 C 是可数个 2 链的并，D 是 C 并上一个独立元素得到的集合。C 和 D 拥有完全相同的茎——任意有限个 2 链与一个有限独立反链的并，因此 C 和 D 是等价反常因果集

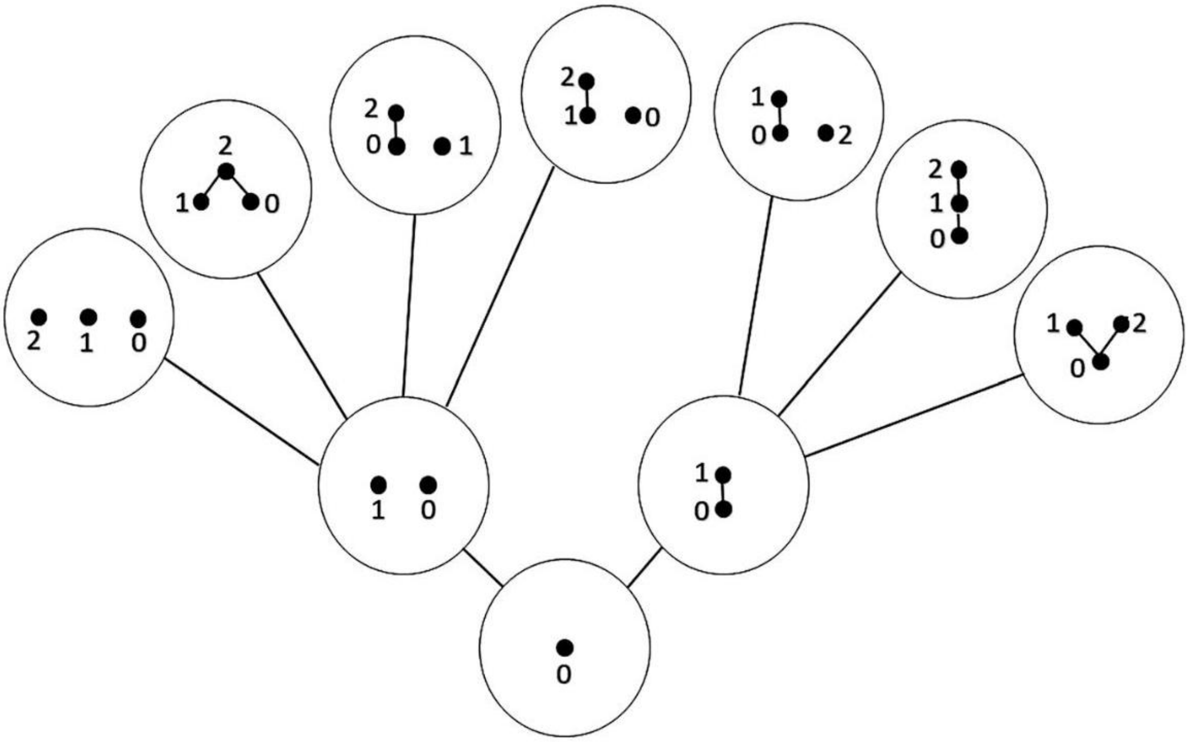


Fig. 4 Sequential growth. Elements are labeled by the stage at which they are born

图 4 顺序增长。元素按其生成的阶段标记

Growth Dynamics

增长动力学

What role do labels play within the growth dynamics framework? In the sequential growth models, elements are born one by one, and so they are labeled by the stage at which they are born (Fig. 4). Thus, each realization is a labeled causet, and the sample space (allowing the process to continue ad infinitum) is the set of infinite labeled causets, $\widetilde{\Omega}$. But our understanding of labels as pure gauge suggests that only the covariant statements that we can make about these realizations are physical. We now make this notion precise.

标签在增长动力学框架中发挥什么作用？在顺序增长模型中，元素逐个生成，因此会按它们生成的阶段进行标记 (图 4)。因此，每个实现都是一个标记因果集，样本空间 (允许过程无限延续) 是无限标记因果集的集合 $\widetilde{\Omega}$ 。但我们将标签理解为纯规范，这表明只有我们能对这些实现做出的协变表述才是物理的。我们现在将这个概念精确化。

For each finite labeled causet $\widetilde{C}_n$, define the "cylinder set,"

对每个有限标记因果集 $\widetilde{C}_n$ ，定义“柱集”，

$$\operatorname{cyl}\left(\widetilde{C}_n\right) := \{\widetilde{C} \in \widetilde{\Omega} \mid \widetilde{C}_n \text{ is a stem in } \widetilde{C}\}. \tag{1}$$

We denote the σ -algebra generated by the cylinder sets by $\widetilde{\mathcal{R}}$. $\left(\widetilde{\Omega}, \widetilde{\mathcal{R}}\right)$ is a measurable space on which each sequential growth model induces a unique probability measure $\widetilde{\mu}$ satisfying,

我们将柱集生成的 σ 代数记为 $\widetilde{\mathcal{R}}$. $\left(\widetilde{\Omega}, \widetilde{\mathcal{R}}\right)$ ，这是一个可测空间，每个顺序增长模型都会在其上诱导出唯一满足如下条件的概率测度 $\widetilde{\mu}$,

$$\widetilde{\mu}\left(\operatorname{cyl}\left(\widetilde{C}_n\right)\right) = \mathbb{P}\left(\widetilde{C}_n\right) \forall \widetilde{C}_n \in \widetilde{\Omega}(\mathbb{N}), \tag{2}$$

where $\mathbb{P}\left(\widetilde{C}_n\right)$ is the model-dependent probability that the first n elements form $\widetilde{C}_n$. The existence of $\widetilde{\mu}$ is guaranteed by the so-called extension theorem of measure theory which states that a pre-measure on a semiring (the cylinder sets form a semiring on which (2) defines a pre-measure) extends to a measure on the σ -algebra generated by the semiring [40].

其中 $\mathbb{P}\left(\widetilde{C}_n\right)$ 是前 n 个元素构成 $\widetilde{C}_n$ 的依赖模型的概率。$\widetilde{\mu}$ 的存在性由测度论中的所谓延拓定理保证，该定理指出，半环上的预测度 (柱集构成一个半环，式 (2) 在该半环上定义了一个预测度) 可以延拓为该半环生成的 σ 代数上的测度 [40]。

Using the terminology of measure theory, each set of histories $\widetilde{\mathcal{E}} \in \widetilde{\mathcal{R}}$ is an event. Physically, the interpretation of these events is as observables or beables. Thus, we are only interested in the covariant events, defined as follows.

用测度论的术语来说，每个历史集合 $\widetilde{\mathcal{E}} \in \widetilde{\mathcal{R}}$ 都是一个事件。物理上，这些事件被解释为可观测量或存在量。因此，我们只对如下定义的协变事件感兴趣。

Definition 11 (Covariant event). An event $\widetilde{\mathcal{E}} \in \widetilde{\mathcal{R}}$ is covariant if whenever $\widetilde{C} \in \widetilde{\mathcal{E}}$ and $\widetilde{C} \cong \widetilde{D}$, then $\widetilde{D} \in \widetilde{\mathcal{E}}$.

定义 11(协变事件)。如果只要 $\widetilde{C} \in \widetilde{\mathcal{E}}$ 和 $\widetilde{C} \cong \widetilde{D}$ 成立，就有 $\widetilde{D} \in \widetilde{\mathcal{E}}$，那么事件 $\widetilde{\mathcal{E}} \in \widetilde{\mathcal{R}}$ 是协变的。

In words, a covariant event is one which cannot distinguish between order-isomorphic causets. The collection of covariant events is a σ -algebra (a sub- σ - algebra of $\widetilde{\mathcal{R}}$), and we denote it by $\mathcal{R}$.

换句话说，协变事件无法区分序同构的因果集。所有协变事件的集合构成一个 σ 代数 (是 $\widetilde{\mathcal{R}}$ 的子 σ 代数)，我们将其记为 $\mathcal{R}$ 。

One can conceive of $\mathcal{R}$ as a σ -algebra on Ω , the set of infinite orders, via the projection $p : \widetilde{\Omega} \to \Omega$ which assigns to each causet $\widetilde{C}$ the order C of which it is a representative. Whether a covariant event is a set of causets or a set of orders is of no consequence for our purposes, and we will use the two interchangeably depending on which is more convenient in the context.

> 我们可以把 $\mathcal{R}$ 看作无限序集合 Ω 上的 σ 代数，这通过投影 $p : \widetilde{\Omega} \to \Omega$ 实现: 该投影给每个因果集 $\widetilde{C}$ 指定它作为代表所属的序 C 。对我们的研究目的而言，协变事件是因果集的集合还是序的集合无关紧要，我们会根据上下文哪个更方便来互换使用这两种表述。

In light of this, one may formulate the problem of covariant dynamics as a two-part question: Can a measure be defined directly on $\mathcal{R}$ (without using $\widetilde{\mathcal{R}}$ as an intermediary)? And if so, can it be done by means of a random walk whose sense of dynamical progression one may interpret as growth? Since one usually considers random walks on finite valency trees which give rise to a countable semiring of cylinder sets, one technical question which underpins this discussion is whether $\mathcal{R}$ is countably generated. As far as the author is aware, this is an open question.

> 有鉴于此，我们可以将协变动力学问题表述为一个两部分问题: 能否直接在 $\mathcal{R}$ 上定义测度 (不以 $\widetilde{\mathcal{R}}$ 作为中介)? 如果可以，能否通过随机游走完成，其动力学进程的含义可以被解释为增长? 由于人们通常考虑有限价树的随机游走，它会产生可数半环的柱集，支撑本次讨论的一个技术问题是 $\mathcal{R}$ 是否可数生成。据作者所知，这是一个开放问题。

Instead, the direction which has been pursued by the community has been to study sub- σ -algebras of $\mathcal{R}$ [12, 13, 15] . This approach has several advantages. First, the sub- σ -algebras of interest have been shown to be isomorphic to the topological σ -algebras of certain trees. This means that one can define a measure on them by means of a random walk on a tree, allowing for a growth dynamics picture. Second, while all the events in $\mathcal{R}$ are covariant, the physical interpretation of these events remains largely obscure. But in some sub- σ -algebras, all events can be assigned a physical interpretation, an argument for considering them alone as the physical set of observables. Third, this kinematic argument for narrowing the set of observables is strengthened by a dynamical one. Consider some measure μ on $\mathcal{R}$ which satisfies $\mu(\mathcal{E}) = 0$ for some $\mathcal{E} \in \mathcal{R}$. Then the measure of an arbitrary event $\mathcal{F} \in \mathcal{R}$ is fixed via $\mu(\mathcal{F}) = \mu(\mathcal{F} \setminus \mathcal{E})$. The collection of events of the form $\mathcal{F} \setminus \mathcal{E}$ is contained in some sub- σ -algebra of $\mathcal{R}$ which we denote as $\mathcal{R}_{\mathcal{E}} \subset \mathcal{R}$. Physically, we can interpret this statement as saying that the events in $\mathcal{R} \setminus \mathcal{R}_{\mathcal{E}}$ contain no new dynamical information and therefore $\mathcal{R}_{\mathcal{E}}$ exhausts the set of observables for the particular dynamics μ .

> 相反，学界目前的研究方向是研究 σ 子代数 ($\mathcal{R}$ [12, 13, 15] 的子代数)。这种方法有诸多优势。首先，已证明我们关注的这类 σ 子代数同构于特定树的拓扑 σ 代数，这意味着我们可以通过树上的随机游走在这类子代数上定义测度，从而得到增长动力学图像。其次，尽管 $\mathcal{R}$ 中的所有事件都是协变的，但这些事件的物理解释大多仍不明确。而在部分 σ 子代数中，所有事件都可以赋予物理解释，这也支持了仅将这类子代数作为物理可观测量集合的观点。第三，这种缩小可观测量集合的运动学论证得到了动力学论证的进一步支撑。考虑 $\mathcal{R}$ 上满足 $\mu(\mathcal{E}) = 0$ (对某个 $\mathcal{E} \in \mathcal{R}$) 的某个测度 μ ， 任意事件 $\mathcal{F} \in \mathcal{R}$ 的测度可通过 $\mu(\mathcal{F}) = \mu(\mathcal{F} \setminus \mathcal{E})$ 确定。所有形如 $\mathcal{F} \setminus \mathcal{E}$ 的事件集合都包含在 $\mathcal{R}$ 的某个 σ 子代数中，我们将该子代数记作 $\mathcal{R}_{\mathcal{E}} \subset \mathcal{R}$ 。从物理上我们可以将该结论解读为: $\mathcal{R} \setminus \mathcal{R}_{\mathcal{E}}$ 中的事件不包含新的动力学信息，因此 $\mathcal{R}_{\mathcal{E}}$ 已经穷尽了特定动力学 μ 的所有可观测量。

The CSG models make a good case-study for all three arguments. Proceeding in reverse order, in the CSG models, the set of all rogues (the event that spacetime is a rogue) has measure zero, and the measure on the stem algebra (defined below) is sufficient to recover the measure on $\mathcal{R}$ [13].

序贯增长模型 (CSG) 为上述三个论证提供了很好的研究案例。我们倒序来看: 在 CSG 模型中，所有反常因果集的集合 (时空为反常因果集的事件) 测度为零，干代数 (下文会定义) 上的测度足以重构 $\mathcal{R}$ 上的测度 [13]。

For each n -order C_n , its stem set is defined as

对任意 n 阶 C_n ，其干集定义为

$$\operatorname{stem}(C_n) := \left\{\widetilde{D} \in \widetilde{\Omega} \mid C_n \text{ is a stem in } \widetilde{D}\right\} \tag{3}$$

and is equal to the union of cylinder sets of the labeled causets in which C_n is a stem. We denote the σ -algebra generated by the stem sets by $\mathcal{R}(\mathcal{S})$ and note that $\mathcal{R}(\mathcal{S}) \subset \mathcal{R}$.

它等于所有带标号因果集的柱集的并，这些因果集中 C_n 是一个干。我们将干集生成的 σ 代数记作 $\mathcal{R}(\mathcal{S})$ ，且有 $\mathcal{R}(\mathcal{S}) \subset \mathcal{R}$.

In fact, one can identify strictly smaller σ -algebras in $\mathcal{R}(\mathcal{S})$ from which the measure on $\mathcal{R}$ can be recovered. But there is a strong kinematic argument for crowning $\mathcal{R}(\mathcal{S})$ as the physical set of observables: each event in $\mathcal{R}(\mathcal{S})$ has a physically meaningful interpretation as a logical combination of statements about which finite orders are stems in the growing causal set (e.g., the event stem(.) $\cap$ stem( ) corresponds to the statement I and . are both stems in the growing causet). Therefore, one can characterize the events in $\mathcal{R}(\mathcal{S})$ as those covariant events which do not distinguish between equivalent rogues.

事实上，我们可以在 $\mathcal{R}(\mathcal{S})$ 中找到更小的 σ 代数，同样可以重构 $\mathcal{R}$ 上的测度。但有强有力的运动学论证支持将 $\mathcal{R}(\mathcal{S})$ 定为物理可观测量集合: $\mathcal{R}(\mathcal{S})$ 中的每个事件都有物理意义，它们是关于哪些有限阶是增长因果集中的干这一命题的逻辑组合 (例如事件 stem(.) $\cap$ stem( ) 对应命题“我和. 都是增长因果集中的干”)。因此，我们可以将 $\mathcal{R}(\mathcal{S})$ 中的事件特征概括为: 不区分等价反常因果集的协变事件。

Finally, can we conceive of a measure on $\mathcal{R}(\mathcal{S})$ in terms of a random walk? The stem sets can be arranged into a partial order by means of set inclusion (i.e., $\operatorname{stem}(C) \prec \operatorname{stem}(D)$ if $\operatorname{stem}(C) \supset \operatorname{stem}(D)$) . This ordering of the stem sets is equivalent to poscau (Fig. 5).

最后，我们能否通过随机游走在 $\mathcal{R}(\mathcal{S})$ 上构造一个测度? 干集可通过集合包含关系构成偏序 (即，若 $\operatorname{stem}(C) \supset \operatorname{stem}(D)$) 则 $\operatorname{stem}(C) \prec \operatorname{stem}(D)$)。干集的这种偏序等价于 poscau(图 5)。

Definition 12 (Poscau). Poscau is a partial order on finite orders, $(\Omega(\mathbb{N}), \prec)$, where $A \prec B$ if and only if A is a stem in B .

定义 12(Poscau) Poscau 是有限序上的偏序 $(\Omega(\mathbb{N}), \prec)$ ，当且仅当 A 是 B 中的一个干时，$A \prec B$ 成立。

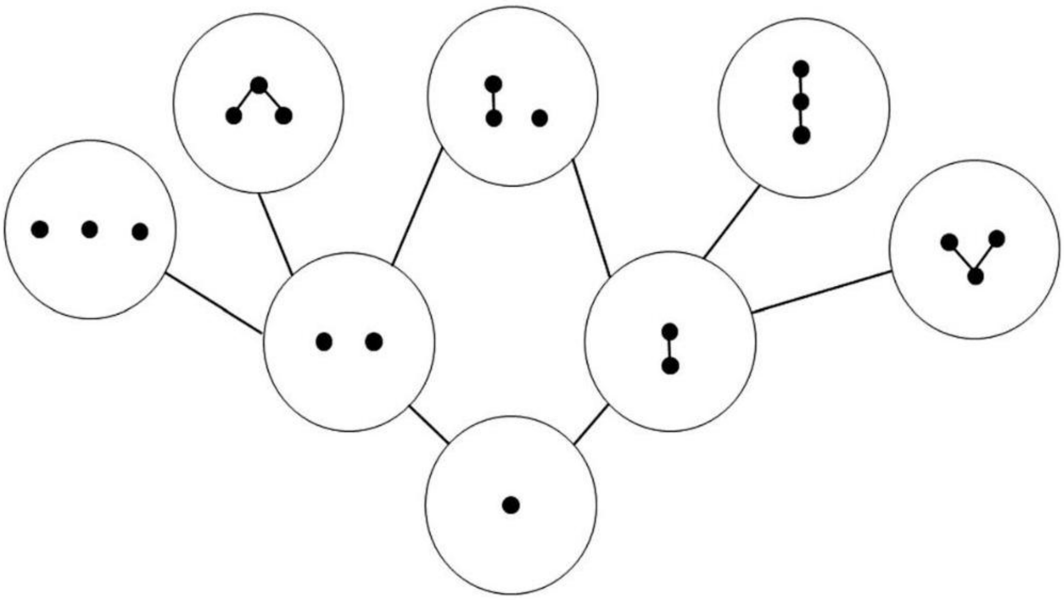


Fig. 5 The first three levels of poscau

图 5 poscau 的前三层

To conceive of a random walk on poscau as a physical process, each node should carry a clear physical meaning. Naively, arriving at the node A corresponds to the occurrence of the covariant event $s\,\text{tem}\,(A)$. But this fails because the physical interpretation of the nodes implies that each growing causal set contains only one n - stem for each $n > 0$ (which is untrue in general). This failure is routed in the fact that poscau is not a tree which is closely related to the fact that the collection of stem sets does not behave like a collection of cylinder sets, since $\text{stem}\,(A) \cap \text{stem}\,(B) \neq \varnothing$ for all A, B .

要将 poscau 上的随机游走构想为一个物理过程，每个节点都必须具备明确的物理意义。直观来看，到达节点 A 对应协变事件 $s\,\text{tem}\,(A)$ 发生。但该观点不成立，因为节点的物理解释要求每个增长的因果集对于任意 $n > 0$ 都仅包含一个 n 干，而一般情况下并非如此。该问题的根源在于 poscau 不是树，这与干集族不具备柱集族的性质密切相关，因为对所有 A, B 都有 $\text{stem}\,(A) \cap \text{stem}\,(B) \neq \varnothing$ 。

Introducing Covtree

介绍 Covtree

We discussed the difficulty of assigning physical meaning to a walk on poscau, and thinking in this way suggests the solution: a covariant dynamics can be defined as a walk on a tree formed of countably many levels in which the nodes in level n are not single n -orders but sets of n -orders. Each set of n -orders in level n will correspond to the covariant event ”the n -stems of the growing causal set are the elements of this set.” We call this tree covtree, short for covariant tree.

我们已经讨论过为偏序因果集上的游走赋予物理意义存在困难，而这种思考方式指明了解决方案: 协变动力学可以定义为可数分层构成的树上的游走，其中第 n 层的节点不是单个 n 阶，而是若干 n 阶的集合。第 n 层中每个 n 阶的集合都对应协变事件“增长中因果集的 n 干就是该集合的元素”。我们将这棵树称为 covtree，是协变树 (covariant tree) 的缩写。

All definitions and results presented in this section are taken from [32].

本节所有定义与结论均引自文献 [32]。

Certificates

证明证书

We now introduce the notion of certificate which will play a key role in the definition of covtree and in its interpretation as a framework for growth dynamics.

我们现在引入证书的概念，它在 covtree 的定义以及将其阐释为增长动力学框架的过程中都发挥着关键作用。

Let $\Gamma_n \subseteq \Omega(n)$ be a non-empty set of n -orders.

设 $\Gamma_n \subseteq \Omega(n)$ 为 n 序的一个非空集合。

Definition 13 (Certificate). A finite or infinite order C is a certificate of Γ_n if Γ_n is the set of all n -stems in C .

定义 13(证书): 若 Γ_n 是 C 中所有 n 干的集合，则有限或无限序 C 是 Γ_n 的一个证书。

Given some Γ_n , it may or may not have a certificate. We will be interested in those Γ_n which do have a certificate.

对于任意给定的 Γ_n ，它可能拥有证书，也可能没有。我们关注的是那些确实拥有证书的 Γ_n 。

Definition 14 (Λ , the collection of certified sets). Λ is the collection of sets of n -orders, for all n , for which there exists a certificate:

定义 14(Λ ，认证集合族): Λ 是对所有 n 而言，所有存在证书的 n 序集合构成的族:

$$\Lambda := \bigcup_{n>0} \{\Gamma_n \subseteq \Omega(n) \mid \exists \text{ a certificate for } \Gamma_n\}. \tag{4}$$

One can show that each $\Gamma_n \in \Lambda$ has infinitely many certificates, including infinitely many finite certificates and infinitely many infinite certificates. We will often work with the minimal certificates:

可以证明，每个 $\Gamma_n \in \Lambda$ 都拥有无穷多个证书，其中包含无穷多个有限证书和无穷多个无限证书。我们通常会研究极小证书:

Definition 15 (Minimal certificate). Given some $\Gamma_n \in \Lambda$, we order its finite certificates as follows: let C, C' be finite certificates of Γ_n , then $C \preccurlyeq C'$ if C is a stem in C' . A minimal certificate of Γ_n is minimal in this partial order of certificates.

定义 15(极小证书): 给定任意 $\Gamma_n \in \Lambda$ ， 我们按如下规则对其有限证书排序: 设 C, C' 是 Γ_n 的有限证书，若 C 是 C' 中的一个干，则 $C \preccurlyeq C'$ 成立。Γ_n 的极小证书是该证书偏序中的极小元。

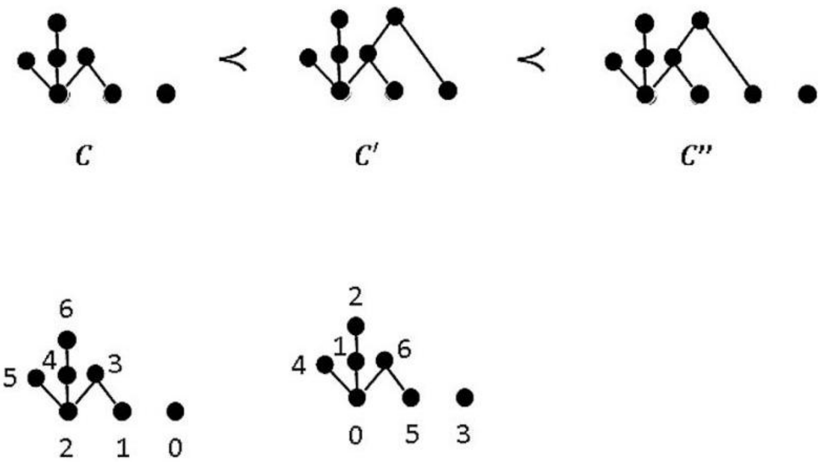


Fig. 6 Certificates. C, C' , and C'' are finite certificates of $\Omega(3)$, the set of all 3-orders. The $\prec$ relation indicates inclusion by stem. C is a minimal certificate of $\Omega(3)$. The labeled causets shown are representatives of C and hence are labeled minimal certificates of $\Omega(3)$

图 6 证书示例。C, C' 和 C'' 是所有 3-阶构成的集合 $\Omega(3)$ 的有限证书。$\prec$ 关系表示通过干的包含关系。C 是 $\Omega(3)$ 的一个极小证书。图中所示的标记因果集是 C 的代表元，因此也是 $\Omega(3)$ 的标记极小证书

At times it may be easier to work with labeled causets rather than with orders. To this end we define the labeled analogue of the certificate.

有时使用标记因果集比直接使用序更方便。为此我们定义证书的标记对应概念。

Definition 16 (Labeled certificate). A labeled certificate of Γ_n is a representative of a certificate of Γ_n . A labeled minimal certificate of Γ_n is a representative of a minimal certificate of Γ_n .

定义 16(标记证书): Γ_n 的标记证书是 Γ_n 的证书的一个代表元。Γ_n 的标记极小证书是 Γ_n 的极小证书的一个代表元。

Illustrations are shown in Fig. 6.

示意图见图 6。

Definition of Covtree

Covtree 的定义

We begin by introducing the map $\mathcal{O}$.

我们首先介绍映射 $\mathcal{O}$ 。

Definition 17 (The map $\mathcal{O}$). For any $n > 1$ and any Γ_n , the map $\mathcal{O}$ takes Γ_n to the set of $(n-1)$ -stems of elements of Γ_n :

定义 17(映射 $\mathcal{O}$)。对任意 $n > 1$ 和任意 Γ_n ，映射 $\mathcal{O}$ 将 Γ_n 映射为 Γ_n 中元素的所有 $(n-1)$ -干构成的集合:

$$\mathcal{O}\left(\Gamma_n\right) := \{B_{n-1} \in \Omega\left(n-1\right) \mid \exists A_n \in \Gamma_n \text{ s.t. } B_{n-1} \text{ is a stem in } A_n\}. \tag{5}$$

An illustration is shown in Fig. 7. The exponentiation $\mathcal{O}^k$ takes Γ_n to the set of $(n-k)$ -stems of elements of Γ_n . If C is a certificate of Γ_n , then C is also a certificate of $\mathcal{O}^k\left(\Gamma_n\right)$ for any $k < n$. (The proof may be summarized by the mnemonic: a stem in a stem is a stem, not a stem in any stem is not a stem) The converse is not true: if C is a certificate of $\mathcal{O}\left(\Gamma_n\right)$, then C may or may not be a certificate of Γ_n (in fact, Γ_n may have no certificates at all).

图 7 给出了示意图。幂运算 $\mathcal{O}^k$ 将 Γ_n 映射为 Γ_n 中元素的所有 $(n-k)$ -干构成的集合。若 C 是 Γ_n 的一个证书，则对任意 $k < n$ ，C 也都是 $\mathcal{O}^k\left(\Gamma_n\right)$ 的证书。(该结论的证明可总结为助记法: 干中的干仍是干，不在任一干中就不是干) 反之不成立: 若 C 是 $\mathcal{O}\left(\Gamma_n\right)$ 的一个证书，C 可能是也可能不是 Γ_n 的证书 (事实上，Γ_n 可能根本没有证书)。

Definition 18 (Covtree). Covtree is the partial order $(\Lambda, \prec)$, where $\Gamma_n \prec \Gamma_m$ if and only if $n < m$ and $\mathcal{O}^{m-n}\left(\Gamma_m\right) = \Gamma_n$.

定义 18(Covtree)。Covtree 是偏序 $(\Lambda, \prec)$ ，其中 $\Gamma_n \prec \Gamma_m$ 当且仅当 $n < m$ 且 $\mathcal{O}^{m-n}\left(\Gamma_m\right) = \Gamma_n$ 。

Fig. 7 Illustration of the map $\mathcal{O}$

图 7 映射 $\mathcal{O}$ 的示意图

$$\mathcal{O}\left(\{, , \cdot, t_0\}\right) = \{1, \ldots\}$$

$$\mathcal{O}\left(\{\text{ Å }, M\}\right) = \{\text{ Å }\}$$

We note some key points about covtree:

我们注意 covtree 的几个关键点:

- Covtree is the partial order on Λ defined by putting each Γ_n directly above $\mathcal{O}\left(\Gamma_n\right)$ and taking the transitive closure. Thus, covtree is a tree.

- Covtree 是定义在 Λ 上的偏序，构造方式为将每个 Γ_n 直接置于 $\mathcal{O}(\Gamma_n)$ 上方后取传递闭包。因此 covtree 是一棵树。

- Covtree has no maximal nodes. Every covtree node is contained in uncountably many inextendible upward-going paths.

- Covtree 没有极大节点。每个 covtree 节点都包含在不可数个不可延伸向上路径中。

- We label the levels of covtree by 1,2,... where level 1 contains the root. The nodes at level n are the sets of n -orders which have certificates (this is the motivation for the term certificate: a certificate of Γ_n certifies that Γ_n is a node in covtree.)

- 我们将 covtree 的层编号为 $1,2,\cdots$，其中第 1 层包含根节点。第 n 层的节点是所有存在证书的 n -序的集合 (这就是“证书”一词的由来: Γ_n 的证书可以证明 Γ_n 是 covtree 中的一个节点)。

- A certificate of a node Γ_n is also a certificate of every node below Γ_n .

- 节点 Γ_n 的证书同时也是 Γ_n 下方所有节点的证书。

- Given a node Γ_n , repeated applications of $\mathcal{O}$ generate the unique path downwards from Γ_n to the root.

给定一个节点 Γ_n ，重复应用 $\mathcal{O}$ 可以得到从 Γ_n 到根的唯一向下路径。

- In order to construct level n of covtree, one considers all the non-empty subsets of $\Omega(n)$. These are the ”candidate nodes” for level n . To determine whether a candidate node is a node in covtree, one needs to determine whether it has a certificate. In general, this is a difficult problem.

要构造 covtree 的第 n 层，我们需要考虑 $\Omega(n)$ 的所有非空子集，这些就是第 n 层的“候选节点”。要判断一个候选节点是否是 covtree 中的节点，需要判断它是否存在证书。一般来说这是一个难题。

- Given any n -order C_n , the set $\{C_n\}$ is a node at level n since C_n is a certificate of $\{C_n\}$.

对任意 n -序 C_n ，集合 $\{C_n\}$ 是第 n 层的一个节点，因为 C_n 就是 $\{C_n\}$ 的证书。

- The first three levels of covtree are shown in Fig. 8. Levels 1 and 2 contain all candidate nodes, while level 3 contains 22 nodes out of 31 candidates. The 9 ”non-nodes” are shown in Fig. 9.

covtree 的前三层如图 8 所示。第 1 层和第 2 层包含所有候选节点，而第 3 层的 31 个候选中仅有 22 个节点。9 个“非节点”如图 9 所示。

The Sample Space, Algebra, and Measure

样本空间、代数与测度

Our definition of certificate implies that each $\widetilde{C} \in \widetilde{\Omega}$ is a labeled certificate of exactly one node at level n , for all $n > 0$. The nodes of which $\widetilde{C}$ is a certificate form a path in covtree $\mathcal{P} = \Gamma_1 \prec \Gamma_2 \prec \dots$, and this allows us to think of $\widetilde{C}$ as a certificate of the path itself:

> 我们对凭证的定义表明，对所有 $n > 0$ ，每个 $\widetilde{C} \in \widetilde{\Omega}$ 恰好是第 n 层一个节点的标记凭证。以 $\widetilde{C}$ 为凭证的节点在协变树 $\mathcal{P} = \Gamma_1 \prec \Gamma_2 \prec \dots$ 中构成一条路径，因此我们可以将 $\widetilde{C}$ 视为该路径本身的凭证:

Definition 19 (Certificate of path). An infinite order C is a certificate of $\mathcal{P}$ if it is a certificate of every node in $\mathcal{P}$. A labeled certificate of $\mathcal{P}$ is a representative of a certificate of $\mathcal{P}$.

> 定义 19(路径凭证)。若无穷阶 C 是 $\mathcal{P}$ 中每个节点的凭证，则它是 $\mathcal{P}$ 的凭证。$\mathcal{P}$ 的标记凭证是 $\mathcal{P}$ 凭证的一个代表。

We note that every $\widetilde{C} \in \widetilde{\Omega}$ is a labeled certificate of exactly one path. This ensures that every $\widetilde{C} \in \widetilde{\Omega}$ is contained in the covtree sample space, where the interpretation

> 我们注意到，每个 $\widetilde{C} \in \widetilde{\Omega}$ 恰好是一条路径的标记凭证。这保证了每个 $\widetilde{C} \in \widetilde{\Omega}$ 都属于协变树的样本空间，其解释为

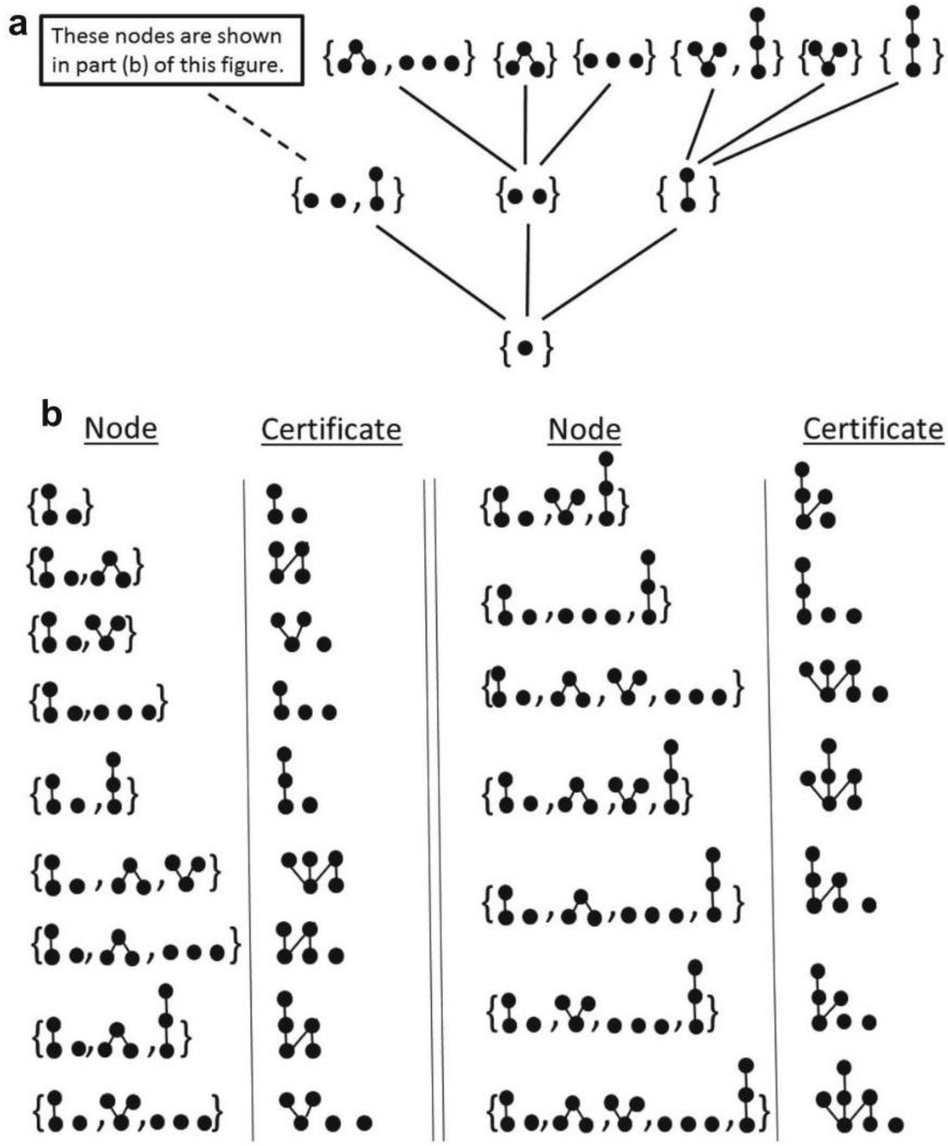


Fig. 8 The first three levels of covtree. (a) The structure of the first three levels of covtree. (b) The level 3 nodes which are directly above the node {1, ...} are shown together with their respective certificates

图 8 协变树的前三层。(a) 协变树前三层的结构。(b) 直接位于节点 {1, ...} 上方的第 3 层节点及其对应凭证

Fig. 9 The sets shown in the figure have no certificates and therefore are not nodes. For every set shown, if an order contains all the elements of that set as stems, then it also contains *.asaktm*

图 9 图中所示集合均无凭证，因此不是节点。对图中每个集合而言，若一个序包含该集合的所有元素作为干，则它也包含 *.asaktm*

$$\{x, x'\}\{x, \xi\}\{\dots, \xi\}\{\dots, \xi\}\{\dots, \xi\}$$

$$\{x, y, \ldots\}\{x, y, z\}\{x, \ldots, y\}$$

$$\{\because, \ldots, i\}, \{\ldots, 0, \ldots, 0\}$$

is that $\widetilde{C}$ is grown by the process when a random walker on covtree picks out the path of which $\widetilde{C}$ is a certificate.

即当协变树上的随机游走选出 $\widetilde{C}$ 作为凭证的路径时，该过程生长出 $\widetilde{C}$ 。

Consider a pair of order-isomorphic causal sets $\widetilde{C}, \widetilde{D} \in \widetilde{\Omega}.\widetilde{C}$ and $\widetilde{D}$ are labeled certificates of the same nodes and are therefore associated with the same path. This is what we expected from a covariant dynamics: order-isomorphic causal sets (i.e., those causal sets which differ from each other only in their labels) cannot be distinguished. Therefore, instead of associating $\mathcal{P}$ with a growing causal set, one can associate $\mathcal{P}$ with a growing order $C = [\widetilde{C}] = [\widetilde{D}]$.

考虑一对序同构的因果集: $\widetilde{C}, \widetilde{D} \in \widetilde{\Omega}.\widetilde{C}$ 和 $\widetilde{D}$ 是相同节点的标记凭证，因此对应同一条路径。这正是我们对协动力学的预期: 序同构的因果集 (即仅标记不同的因果集) 是不可区分的。因此，我们无需将 $\mathcal{P}$ 关联到生长中的因果集，而是可以将 $\mathcal{P}$ 关联到生长中的序 $C = [\widetilde{C}] = [\widetilde{D}]$ 。

When $\widetilde{C}$ is a rogue, the class of causal sets associated with $\mathcal{P}$ is strictly larger than $[\widetilde{C}]$, since if $\widetilde{C}$ and $\widetilde{E}$ form a rogue pair then they are labeled certificates of the same nodes. This inability of covtree to distinguish between equivalent rogues suggests that the covariant σ -algebra on which a random walk on covtree defines a measure is the stem algebra, $\mathcal{R}(\mathcal{S})$, which was introduced in section ”Growth Dynamics”. We shall see that this intuition is correct.

当 $\widetilde{C}$ 是流氓对的一员时，与 $\mathcal{P}$ 关联的因果集严格包含 $[\widetilde{C}]$ ，因为若 $\widetilde{C}$ 和 $\widetilde{E}$ 构成流氓对，则它们是相同节点的标记凭证。协变树无法区分等价流氓的这一性质表明，协变树上定义随机游走测度的协变 σ -代数正是干代数 $\mathcal{R}(\mathcal{S})$ ，我们已在“生长动力学”一节引入该代数。后文将证明这一直觉是正确的。

Before proceeding to consider the σ -algebra and measure in more detail, we must first satisfy ourselves that our interpretation of the nodes as sets of stems allows for every inextendible upward-going path $\mathcal{P}$ to be associated with some $\widetilde{C} \in \widetilde{\Omega}$. This is desirable for two reasons, interpretational and technical: it ensures that every realization of the random walk can be associated with a growing causal set, and it guarantees that every set of transition probabilities on covtree yields a well-defined measure on the covariant event algebra.

在进一步讨论 σ -代数与测度之前，我们首先需要确认: 将节点解释为干的集合确实能保证每条不可延拓向上路径 $\mathcal{P}$ 都对应某个 $\widetilde{C} \in \widetilde{\Omega}$ 。这一点无论是从解释层面还是技术层面都是必要的: 它保证随机游走的每一次实现都能对应一个生长中的因果集, 也保证协变树上的任意一组转移概率都能在协变事件代数上定义一个良定的测度。

Let $\mathcal{P}$ denote an inextendible covtree path from the origin upwards, $\Gamma_1 \prec \Gamma_2 \prec \ldots$

设 $\mathcal{P}$ 为从原点出发向上的不可延拓协变树路径，$\Gamma_1 \prec \Gamma_2 \prec \ldots$

Theorem 1. Every path $\mathcal{P}$ has at least one certificate.

定理 1。每条路径 $\mathcal{P}$ 都至少有一个凭证。

Sketch of proof. We will use the fact that for any $\Gamma_m \in \mathcal{P}$, there exists some $n > m$ such that $\Gamma_n \in \mathcal{P}$ contains some certificate C_n of Γ_m (see lemma 4.4 in [32]).

证明概要。我们将利用如下事实: 对任意 $\Gamma_m \in \mathcal{P}$ ， 存在某个 $n > m$ 使得 $\Gamma_n \in \mathcal{P}$ 包含 Γ_m 的某个证明证书 C_n (参见文献 [32] 中的引理 4.4)。

Choose any $\Gamma_l \in \mathcal{P}$ to begin with.

首先任选一个 $\Gamma_l \in \mathcal{P}$ 。

Pick some $\Gamma_m \in \mathcal{P}$ which contains a certificate C_m of Γ_l . Pick a labeled representative $\widetilde{C}_m$ of C_m .

选取某个包含 Γ_l 的证明证书 C_m 的 $\Gamma_m \in \mathcal{P}$ 。选取 C_m 的一个带标记代表元 $\widetilde{C}_m$ 。

Pick some $\Gamma_n \in \mathcal{P}$ which contains a certificate C_n of Γ_m . Pick a labeled representative $\widetilde{C}_n$ of C_n in which $\widetilde{C}_m$ is a stem. This is always possible because C_m is a stem in C_n .

选取某个包含 Γ_m 的证明证书 C_n 的 $\Gamma_n \in \mathcal{P}$ 。选取 C_n 的一个带标记代表元 $\widetilde{C}_n$ ， 其中 $\widetilde{C}_m$ 是一个干。这总是可行的，因为 C_m 本身就是 C_n 中的干。

Continue iteratively as above, at each stage picking a node which contains a certificate of the previous node and then picking a representative of this certificate with a labeling compatible with the previous labeled certificate.

按照上述方式持续迭代，每一步都选取一个包含前一节点证明证书的节点，随后选取该证书的一个代表元，使其标记与前一个带标记证书相容。

This algorithm produces a countable sequence of labeled causets $\widetilde{C}_m \subset \widetilde{C}_n \subset \ldots$ whose union is a labeled certificate $\widetilde{C}$ of $\mathcal{P}$. The order C , of which $\widetilde{C}$ is a representative, is a certificate of $\mathcal{P}$.

该算法生成一个可数的带标记因果集序列 $\widetilde{C}_m \subset \widetilde{C}_n \subset \ldots$ ， 它们的并是 $\mathcal{P}$ 的一个带标记证明证书 $\widetilde{C}$ 。以 $\widetilde{C}$ 为代表元的序 C 就是 $\mathcal{P}$ 的证明证书。

This establishes the existence of a surjection from Ω , the set of infinite orders, to the set of covtree paths. The upshot is that any realization of a random walk on covtree can be identified with some history in Ω - the growing order is a certificate of the path traced by the random walk.

这就证明了从无穷序集 Ω 到 covtree 路径集之间存在满射。结论是，covtree 上随机游走的任何实现都可以等同于 Ω 中的某个历史——增长的序就是随机游走所追踪路径的证明证书。

As we already mentioned, a path will have more than one certificate if it is associated with rogue orders. How one should resolve this depends on the physical interpretation that one assigns to rogues. If one believes that rogues are unphysical and should never be grown by the process (reasons to think this include that in the

CSG models, rogues never happen, i.e., $\mu(\Theta) = 0$ where Θ is the set of rogues, and that every rogue contains an infinite antichain corresponding to infinite space [13].), then the resolution can be to only consider random walks in which the measure of the set of the "rogue paths" is null. An alternative is to allow rogues to arise but propose that they are physically indistinguishable (since they can only be distinguished globally, not by any local observer living on them). This is equivalent to replacing Ω with the space of rogue equivalence classes $\Omega/\sim_R$, where each path corresponds to exactly one class.

> 如我们此前所述，若一条路径与反常序关联，则它会对应多个证明证书。如何解决这个问题取决于人们对反常序的物理解释。如果认为反常序是非物理的，不该由增长过程产生 (支持该观点的理由包括: 在 CSG 模型中反常序绝不会出现，即当 $\mu(\Theta) = 0$ 中 Θ 是反常序集时，且每个反常序都对应无穷空间包含一个无穷反链 [13])，那么解决方法可以是只考虑“反常路径”集测度为零的随机游走。另一种观点是允许反常序存在，但认为它们在物理上不可区分 (因为只能通过全局性质区分它们，生活在其上的任何局域观察者都无法做到这点)。这等价于将 Ω 替换为反常等价类空间 $\Omega/\sim_R$ ，其中每条路径恰好对应一个等价类。

Another subtlety relates to the notion of growth. To what extent can we say that an order is growing as the covtree walk advances? At stage n , we do not know which finite order has grown thus far nor its cardinality, only which n -stems it contains. While in the CSG models the growth is explicit, on covtree it is implicit or "vague" [20]. But if there is a process of growth which can be associated with a covtree walk, then it may be that it is this quality of vagueness which embodies asynchronous becoming.

> 另一个微妙之处和增长的概念有关。当 covtree 随机游走推进时，我们能在多大程度上说序在增长? 在阶段 n ，我们既不知道目前已经增长出的有限序是什么，也不知道它的基数，只知道它包含哪些 n 干。CSG 模型中增长是显式的，而 covtree 中的增长是隐式的或称“模糊的”[20]。但如果存在一个可与 covtree 随机游走关联的增长过程，那么这种模糊性可能恰恰体现了异步生成。

The surjection from Ω to the set of covtree paths does more than establish a narrative of growth. It enables us to use covtree to define a measure space of orders in the following way. To each covtree node, assign its "cylinder set", the set of all paths $\mathcal{P}$ which contain it. ("Cylinder set" is a generic term in stochastic processes and should not be confused with its specific usage in (1). The meaning should be clear from the context.) The collection of all cylinder sets generates covtree's topological σ -algebra. (It is called "topological" because the cylinder sets are the open balls under the metric topology given by the metric $d(\mathcal{P}, \mathcal{P}') = 1/2^n$, where n is the number of nodes shared by $\mathcal{P}$ and $\mathcal{P}'$.) Now, use the surjection which maps an infinite order to the path of which it is a certificate to pull back covtree's topological σ -algebra to a σ -algebra on Ω . This pull-back algebra is the σ -algebra of observables in a covtree growth dynamics. The pull-back of the cylinder set associated with a given node Γ_n is the set $\mathrm{cert}(\Gamma_n)$, defined by,

> 从 Ω 到协树路径集合的满射不仅仅是构建了增长的描述，它还能让我们通过如下方式利用协树定义序的测度空间: 对每个协树节点，分配其“柱集”，即所有包含该节点的路径 $\mathcal{P}$ 构成的集合。(“柱集”是随机过程中的通用术语，请勿和它在 (1) 中的特定用法混淆，具体含义可通过上下文明确。) 所有柱集的集合生成协树的拓扑 $\sigma\sigma$ 代数。称之为“拓扑”是因为在由度量 $d(\mathcal{P}, \mathcal{P}') = 1/2^n$ 给出的度量拓扑下，柱集就是开球，其中 n 是 $\mathcal{P}$ 和 $\mathcal{P}'$ 共有的节点数量。现在，利用将无穷序映射到其作为证书对应路径的满射，把协树的拓扑 $\sigma\sigma$ 代数拉回为 Ω 上的 $\sigma\sigma$ 代数。这个拉回得到的 σ 代数就是协树增长动力学中可观测量的 $\sigma\sigma$ 代数。和给定节点 Γ_n 关联的柱集的拉回就是集合 $\mathrm{cert}(\Gamma_n)$ ，其定义为,

Definition 20 (Certificate set). For each covtree node Γ_n , its certificate set, $\mathrm{cert}(\Gamma_n)$, is the set containing all its infinite certificates,

<id> 定义 20(证书集)。对每个协树节点 Γ_n ， 它的证书集 $\mathrm{cert}(\Gamma_n)$ 是包含其所有无穷证书的集合,

$$\mathrm{cert}(\Gamma_n) := \{C \in \Omega \mid C \text{ is a certificate of } \Gamma_n\}. \tag{6}$$

Thus, the σ -algebra of observables in the covtree growth dynamics is generated by the certificate sets. In our earlier discussion, we had already anticipated that this σ - algebra is $\mathcal{R}(\mathcal{S})$. Indeed, one can show that any stem set (cf. Equation (3)) can be constructed through a finite number of set operations on the certificate sets and vice versa. It follows that the collection of stem sets and the collection of certificate sets generate the same σ -algebra.

因此，协树增长动力学中可观测量的 σ σ 代数由证书集生成。在我们之前的讨论中，已经预判该 σ σ 代数就是 $\mathcal{R}(\mathcal{S})$ 。事实上，可以证明任何干集 (参见式 (3)) 都可以通过证书集的有限次集合运算构造得到，反之亦然，由此可知干集族和证书集族生成同一个 σ σ 代数。

Finally, standard results in measure theory ensure that each covtree random walk (defined by a complete set of covtree transition probabilities) gives rise to a unique measure on $\mathcal{R}(\mathcal{S})$, where the measure of $\mathrm{cert}(\Gamma_n) \in \mathcal{R}(\mathcal{S})$ is equal to the probability of reaching Γ_n (i.e., to the product of transition probabilities on the path from the root to Γ_n).

最后，测度论中的标准结果保证，每个协树随机游走 (由完整的协树转移概率集定义) 都会在 $\mathcal{R}(\mathcal{S})$ 上诱导出唯一测度，其中 $\mathrm{cert}(\Gamma_n) \in \mathcal{R}(\mathcal{S})$ 的测度等于到达 Γ_n 的概率，即等于从根节点到 Γ_n 路径上转移概率的乘积。

We had seen that any random walk on covtree gives a well-defined measure space of causal sets, and this completes our justification for interpreting covtree as a framework for growth dynamics.

我们已经看到，协树上的任意随机游走都能给出定义良好的因果集测度空间，这就完整证明了将协树解释为增长动力学框架的合理性。

We now have two methods for defining measures on $\mathcal{R}(\mathcal{S})$: via a restriction of a measure $\widetilde{\mu}$ on the labeled σ -algebra $\widetilde{\mathcal{R}}$, where $\widetilde{\mu}$ arises from a random walk on labeled poscau (shown in Fig. 4) (the random walk need not be a CSG model nor must the transition probabilities satisfy any physical conditions.), or directly via a covtree random walk. It has been shown that the two methods give rise to the same class of measures, namely, the class of measure on $\mathcal{R}(\mathcal{S})$. Every measure on $\mathcal{R}(\mathcal{S})$ can be derived from a covtree walk: the transition probability in the covtree walk from node Γ_n to the node Γ_{n+1} directly above it is the measure of $\mathrm{cert}(\Gamma_{n+1})$ divided by the measure of $\mathrm{cert}(\Gamma_{n+1})$. Additionally, every measure on $\mathcal{R}(\mathcal{S})$ possesses some (not necessarily unique) extension to $\widetilde{\mathcal{R}}$ (see lemma 4.9 in [32]), meaning that every measure on $\mathcal{R}(\mathcal{S})$ can be obtained via a restriction of some $\widetilde{\mu}$. Thus, for every walk on labeled poscau - whether it satisfies discrete general covariance or not - there exists a covtree walk which produces the same measure on $\mathcal{R}(\mathcal{S})$. There is no easy relationship between the discrete general covariance condition on a labeled poscau walk and the manifest covariance of a covtree walk.

我们现在有两种方法在 $\mathcal{R}(\mathcal{S})$ 上定义测度: 一种是通过限制标记 σ -代数 $\widetilde{\mathcal{R}}$ 上的测度 $\widetilde{\mu}$ 得到，其中 $\widetilde{\mu}$ 源自标记 poscau 上的随机游走 (如图 4 所示)(该随机游走不必是 CSG 模型，转移概率也不必满足任何物理条件)；另一种是直接通过共树 (covtree) 随机游走定义。已经证明，这两种方法得到的是同一类测度，即 $\mathcal{R}(\mathcal{S})$ 上的所有测度。$\mathcal{R}(\mathcal{S})$ 上的每个测度都可以由共树随机游走推导得到: 在共树随机游走中，从节点 Γ_n 到其直接上方节点 Γ_{n+1} 的转移概率等于 cert (Γ_{n+1}) 的测度除以 cert (Γ_{n+1}) 的测度。此外，$\mathcal{R}(\mathcal{S})$ 上的每个测度都存在某种 (不必唯一) 到 $\widetilde{\mathcal{R}}$ 的延拓 (参见文献 [32] 的引理 4.9)，这意味着 $\mathcal{R}(\mathcal{S})$ 上的每个测度都可以通过限制某个 $\widetilde{\mu}$ 得到。因此，对于标记 poscau 上的任意随机游走——无论它是否满足离散广义协方差——都存在一个共树随机游走，能在 $\mathcal{R}(\mathcal{S})$ 上生成相同的测度。标记 poscau 游走的离散广义协方差条件与共树游走的显式协方差之间不存在简单关联。

The Structure of Covtree

协树的结构

In section ”The Sample Space, Algebra, and Measure”, we had seen that a covtree walk is equivalent to a measure on $\mathcal{R}(\mathcal{S})$ and as such is a dynamics for causal sets. But there is no reason to expect that a generic covtree walk gives rise to a physically interesting dynamics: the class of covtree walks (or equivalently, the class of measures on $\mathcal{R}(\mathcal{S})$) is too vast to be interesting. We need physically motivated conditions to restrict the models to a sub-class worth studying.

在“样本空间、代数与测度”一节中，我们已经知道协树游走等价于 $\mathcal{R}(\mathcal{S})$ 上的一个测度，因此它是因果集的一种动力学。但没有理由认为任意协树游走都能产生具有物理研究价值的动力学: 协树游走的集合 (等价地说，$\mathcal{R}(\mathcal{S})$ 上测度的集合) 太过宽泛，不具备研究意义。我们需要基于物理动机的条件，将模型限制到值得研究的子类中。

The CSG models were derived by posing and solving two such conditions, and it is natural to consider how these conditions could be adapted to the covtree framework. However, when doing so, one comes across an obstacle: the formulation of the conditions satisfied by the CSG models relies on the use of labels to the extent that their potential generalizations to a label-free framework are obscured. This may be expected of the discrete general covariance condition, since its role - to impose invariance under relabeling - is redundant in a framework which makes no reference to labels. But as we saw in section ”The Sample Space, Algebra, and Measure”, the manifest covariance of covtree is not equivalent to the discrete general covariance condition, and what form discrete general covariance takes on the covtree transition probabilities is an interesting open question. The local causality condition satisfied by the CSG models (known as ”Bell causality”) states that the probability of transition from $\widetilde{C}_n$ to one of its children $\widetilde{C}_{n+1}$ depends only on the past the newborn element. The issue there is that one has to pinpoint the new-born element, an impossible task when the objects considered are orders, not causal sets. As it stands, this tension between the global nature of label-independent objects and the local nature of causality is still in need of a resolve. We will return to it briefly in section ”Covtree and Causal Set Cosmology”.

CSG 模型是通过提出并求解两个这类条件推导得到的，因此自然可以思考如何将这些条件适配到协树框架中。然而，适配过程中会遇到一个障碍:CSG 模型满足的条件的表述高度依赖标签的使用，导致它们推广到无标签框架的可能性被掩盖了。这对于离散广义协变条件而言是可预见的，因为它的作用——施加重标记下的不变性——在不引用标签的框架中是多余的。但正如我们在“样本空间、代数与测度”一节中看到的，协树的明显协变性不等价于离散广义协变条件，离散广义协变在协树转移概率中取何种形式是一个值得研究的开放问题。CSG 模型满足的定域因果性条件 (即“贝尔因果性”) 指出，从 $\widetilde{C}_n$ 到其子节点 $\widetilde{C}_{n+1}$ 的转移概率仅依赖于新生元素的过去。这里的问题在于，必须先确定出新生元素，而当我们研究的对象是序而非因果集时，这是无法完成的。就目前来看，无标签对象的全局性与因果性的定域性之间的这种张力仍有待解决。我们会在“协树与因果集宇宙学”一节中简要回述这个问题。

A complementary approach to identifying physical dynamics is requiring that the dynamics favor the physical kinematics (e.g., requiring that manifold-like orders are likely to be grown). (We say an order C is manifold-like if a representative of C can be faithfully embedded into a four-dimensional Lorentzian manifold.) The success of translating such requirements into conditions on covtree transition probabilities hinges on understanding the relationship between paths and their certificates (e.g., which paths have manifold-like certificates).

识别物理动力学的另一种互补方法是要求动力学偏好物理运动学 (例如，要求类流形序更有可能被生长出来)。(当序 C 的一个代表元可以忠实嵌入四维洛伦兹流形时，我们称该序 C 是类流形的。) 能否成功将这类要求转化为协树转移概率的约束条件，取决于对路径与其证书之间关系的理解 (例如，哪些路径拥有类流形证书)。

An understanding of the structure of covtree is also important for constraining the dynamics. For example, any dynamics should satisfy the Markov-sum-rule: the sum of the transition probabilities from any node Γ_n must equal 1. But with no knowledge of the number of nodes directly above Γ_n or of the relation they bear to it, this constraint is intractable. (In contrast, in the case of the CSG models knowing that the children of $\widetilde{C}_n$ in labeled poscau are in 1-to-1 correspondence with the stems in $\widetilde{C}_n$ allows to solve the Markov-sum-rule.)

理解协树的结构对于约束动力学也很重要。例如，任何动力学都必须满足马尔可夫求和规则: 任意节点 Γ_n 的转移概率之和必须等于 1。但如果不知道 Γ_n 直接上层节点的数量，也不知道它们和 Γ_n 的关系，这个约束就无法处理。(对比来看，在 CSG 模型中，我们知道标记偏序因果集中 $\widetilde{C}_n$ 的子节点与 $\widetilde{C}_n$ 中的茎一一对应，这就可以求解马尔可夫求和规则。)

In addressing these challenges, one might be tempted to construct covtree explicitly, but thus far only the first three levels of covtree have been worked out (Fig. 8). Brute force methods come up short in going to higher levels as the number of candidate nodes at level n increases rapidly as $2^{|\Omega(n)|}-1$, where $|\Omega(3)| = 5, |\Omega(5)| = 63$, and $|\Omega(16)| = 4483130665195087$ [41]. However, progress has been made by focusing on structural properties which are independent of level. This section is dedicated to surveying these results. The interested reader may refer to [33] for their derivation.

面对这些挑战，人们很容易想到直接显式构造协树，但到目前为止，我们只得到了协树的前三层 (图 8)。暴力方法无法推广到更高层级，因为层级 n 的候选节点数量会随 $2^{|\Omega(n)|}-1$ 快速增长，其中 $|\Omega(3)| = 5, |\Omega(5)| = 63$ ， 以及 $|\Omega(16)| = 4483130665195087$ [41]。不过，聚焦于不依赖层级的结构性质已经取得了进展。本节将综述这些结果，感兴趣的读者可以查阅文献 [33] 了解推导过程。

Nodes

节点

Here, we list properties which pertain to nodes, including criteria for a set of n -orders to be a node, properties of minimal certificates, and a study of direct descendants and valency. We begin with definitions.

在此，我们列出与节点相关的性质，包括一组 n -阶成为节点的判据、最小证书的性质，以及对直接后代和价的研究。我们先从定义开始。

Definition 21 (Singleton and doublet). A node Γ_n in covtree is a singleton if it contains a single n -order. A node Γ_n in covtree is a doublet if it contains exactly two n -orders.

定义 21(单元素与双元素): 协树中的节点 Γ_n 若仅包含一个 n -阶，则为单元素节点。协树中的节点 Γ_n 若恰好包含两个 n -阶，则为双元素节点。

Definition 22 (Covering causet/order). Given an n -causet $\widetilde{C}_n$, its covering causet $\widehat{\widetilde{C}}_n$ is the $(n+1)$ -causet formed by putting the element n above every element of $\widetilde{C}_n$. Similarly, $\widehat{C_n}$ is the covering order of C_n , where $\widehat{\widetilde{C}}_n$ and $\widetilde{C}_n$ are representatives of the respective orders.

定义 22(覆盖因果集/阶): 给定一个 n 因果集 $\widetilde{C}_n$ ， 其覆盖因果集 $\widehat{\widetilde{C}}_n$ 是将元素 n 置于 $\widetilde{C}_n$ 的每个元素之上得到的 $(n+1)$ 因果集。类似地，$\widehat{C_n}$ 是 C_n 的覆盖阶，其中 $\widehat{\widetilde{C}}_n$ 和 $\widetilde{C}_n$ 分别是对应阶的代表元。

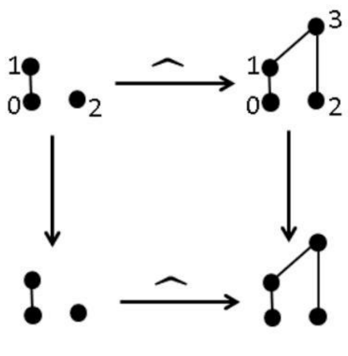


Fig. 10 The relationship

图 10 关系

between an order, its covering

一个阶与其覆盖

order, and their

阶，以及它们的

representatives

代表元

An illustration is shown in Fig. 10.

示例如图 10 所示。

Note that C_n is the only n -stem in its covering order $\widehat{C_n}$, and therefore the node $\{C_n\}$ is directly above $\{C_n\}$ in covtree. Thus,

注意，C_n 是其覆盖阶 $\widehat{C_n}$ 中唯一的 n 干，因此节点 $\{C_n\}$ 在协树中直接位于 $\{C_n\}$ 上方。由此可得:

Property 1. Every singleton has at least one direct descendant which is a singleton.

性质 1 每个单元素节点至少存在一个同为单节点的直接后代。

Moreover,

此外,

Property 2. If $\left\{\widehat{C_n}\right\}$ is the only singleton directly above $\{C_n\}$, then $\left\{\widehat{C_n}\right\}$ is the only node directly above $\{C_n\}$.

性质 2 若 $\left\{\widehat{C_n}\right\}$ 是直接位于 $\{C_n\}$ 上方的唯一单元素节点，则 $\left\{\widehat{C_n}\right\}$ 也是直接位于 $\{C_n\}$ 上方的唯一节点。

Every singleton with valency greater than one has at least one direct descendant which is a doublet since:

每个价大于 1 的单元素节点至少存在一个同为双元素节点的直接后代，原因如下:

Property 3. If $\{D_{n+1}\} \succ \{C_n\}$ and $D_{n+1} \neq \widehat{C_n}$ then $\left\{\widehat{C_n}, D_{n+1}\right\} \succ \{C_n\}$.

性质 3 若 $\{D_{n+1}\} \succ \{C_n\}$ 且 $D_{n+1} \neq \widehat{C_n}$ ，则 $\left\{\widehat{C_n}, D_{n+1}\right\} \succ \{C_n\}$ 。

A corollary of Properties 2 and 3 is:

由性质 2 和性质 3 可推得如下推论:

Property 4. No singleton has a valency of 2.

性质 4: 没有单元素节点的价为 2。

Singletons which possess Property 2 are the only nodes in covtree which have exactly one direct descendant since:

满足性质 2 的单元素节点是协变树中仅有的恰好拥有一个直接后代的节点，原因如下:

Property 5. Only singletons can have exactly one direct descendant in covtree.

性质 5: 只有单元素节点可以在协变树中恰好拥有一个直接后代。

Additionally,

此外，

Property 6. For any $k \geq 1$, there is a singleton $\{C_n\}$ in covtree with k singletons directly above it.

性质 6: 对任意 $k \geq 1$ ，协变树中存在一个单元素节点 $\{C_n\}$ ，有 k 个单元素节点直接在其上方。

An immediate corollary of Property 6 is that the valency of singletons is unbounded. (Note that k is not the valency of $\{C_n\}$, for if $k \geq 2$, then $\{C_n\}$ has additional direct descendants which are not singletons; cf. Property 3.)

性质 6 的一个直接推论是单元素节点的价无界。(注意 k 不是 $\{C_n\}$ 的价，因为若 $k \geq 2$ ，则 $\{C_n\}$ 还有额外的非单元素直接后代；参见性质 3。)

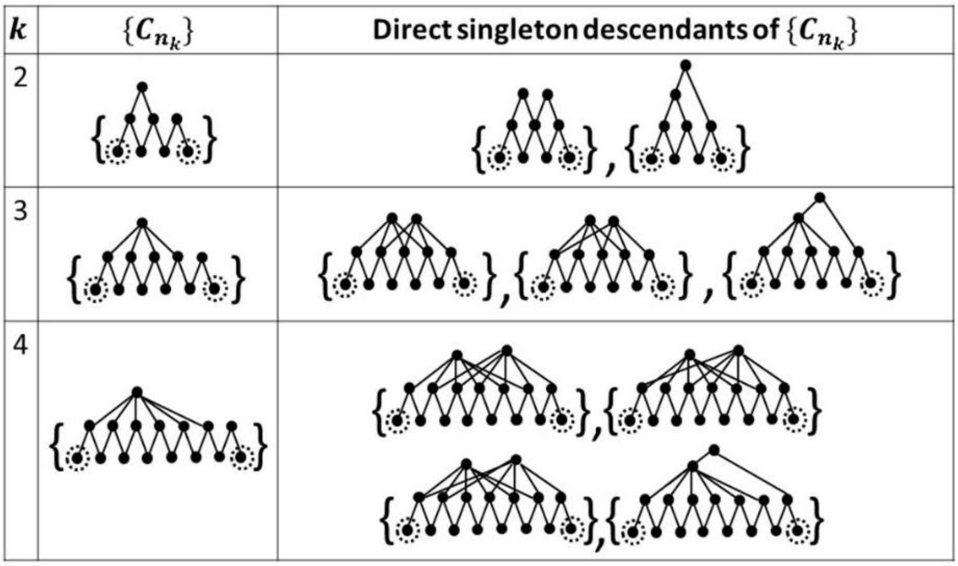


Fig. 11 Illustration of Property 6. The elements circled by a dotted line are identified with each other

图 11 性质 6 的示意图。虚线圆圈圈出的元素彼此等同

An example of a singleton node with 1 singleton directly above it is $\Gamma_4 = \{[]\}$. To see that the statement is true for $k > 1$, one can construct a countable sequence of singletons

有 1 个单元素节点直接在其上方的单元素节点示例为 $\Gamma_4 = \{[]\}$ 。要证明该命题对 $k > 1$ 成立，可以构造一个可数的单元素序列

$$\{C_{n_2}\}, \{C_{n_3}\}, \ldots, \{C_{n_k}\}, \ldots$$

such that $\{C_{n_k}\}$ has k singletons directly above it. Figure 11 shows the first three singletons in the sequence and their respective singleton descendants.

使得 $\{C_{n_k}\}$ 拥有 k 个直接在其上方的单元素节点。图 11 展示了该序列中的前三个单元素节点，以及它们各自的单元素后代。

Similarly,

类似地,

Property 7. For any integer $k \geq 1$, there exists a doublet in covtree with k singletons directly above it.

性质 7: 对任意整数 $k \geq 1$ ，协变树中存在一个双元素节点，有 k 个单元素节点直接在其上方。

As before, one can construct an countable sequence of doublets,

同之前的操作一样，可以构造一个可数的双元素序列,

$$\{C_{m_1}, D_{m_1}\}, \{C_{m_2}, D_{m_2}\}, \ldots, \{C_{m_k}, D_{m_k}\}, \ldots$$

such that the k th doublet in the sequence has k singletons directly above it. Figure 12 shows the first three doublets in the sequence and their direct singleton descendants.

使得序列中第 k 个双元素节点拥有 k 个直接在其上方的单元素节点。图 12 展示了该序列中的前三个双元素节点，以及它们的直接单元素后代。

A key hurdle in the construction of covtree is understanding which sets of n - orders are covtree nodes. The following property gives a necessary condition in the case of doublets:

构造协变树的一个核心难点是弄清哪些 n 序集合是协变树节点。下述性质给出了双元素节点情况下的必要条件:

Property 8. $\{A_n, B_n\}$ is a doublet in covtree only if there exists an $(n-1)$ -order S which is a stem in both A_n and B_n .

性质 8: $\{A_n, B_n\}$ 是协变树中的双元素节点，仅当存在一个 $(n-1)$ 序 S ，它同时是 A_n 和 B_n 中的干。

Property 9 is a corollary:

性质 9 是一个推论:

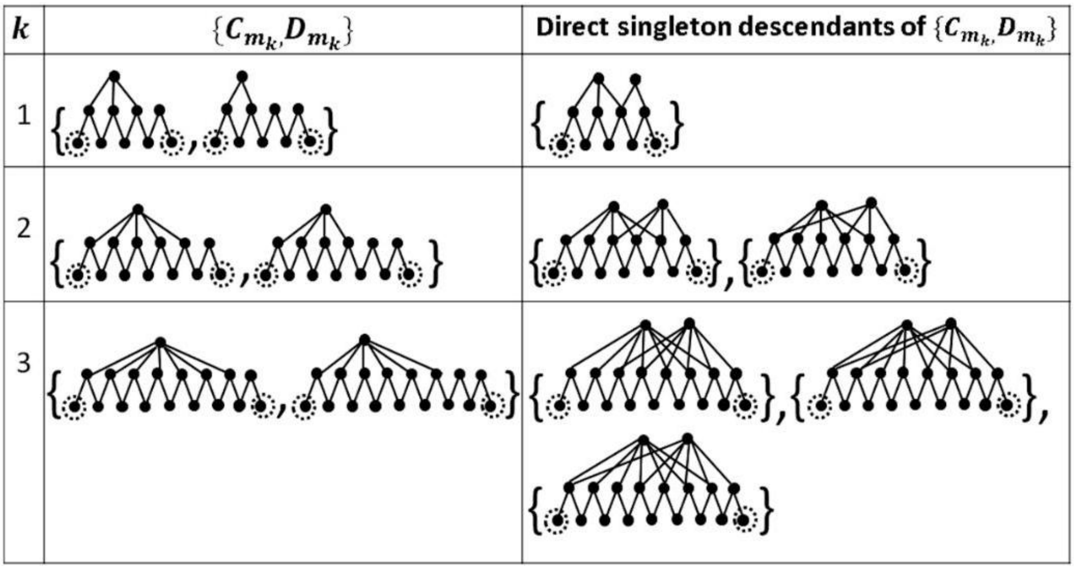
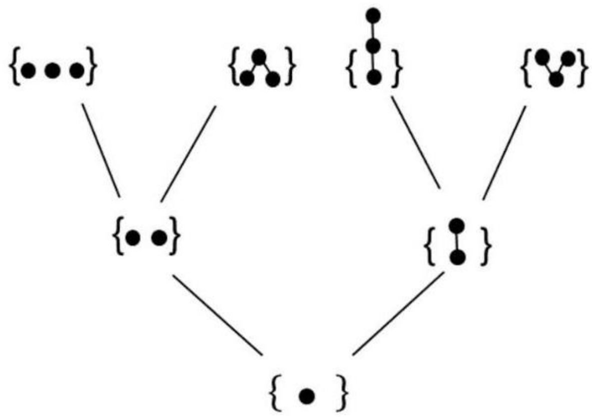


Fig. 12 Illustration of Property 7. The elements circled by a dotted line are identified with each other

图 12 性质 7 的图示。虚线圆圈圈出的元素彼此等同

Fig. 13 The first three levels of singtree

图 13 单点树的前三层结构



Property 9. If Γ_n is a doublet in covtree, then all minimal certificates of Γ_n are $(n+1)$ -orders.

性质 9。若 Γ_n 是协树中的二元组，则 Γ_n 的所有极小证书都是 $(n+1)$ 序。

Therefore, if Γ_n is a doublet in covtree and $\Gamma_n \prec \Gamma_{n+1}$, then Γ_{n+1} contains some minimal certificate of Γ_n . It is a corollary of Properties 7 and 9 that for any integer $k \geq 1$ there exists a doublet in covtree with k minimal certificates.

因此，若 Γ_n 是协树中的二元组且满足 $\Gamma_n \prec \Gamma_{n+1}$ ，则 Γ_{n+1} 包含 Γ_n 的某个极小证书。由性质 7 和性质 9 可推得: 对任意整数 $k \geq 1$ ，协树中存在一个拥有 k 个极小证书的二元组。

Paths

路径

Here, we present properties of certain covtree paths and their certificates.

在此，我们介绍协变树中特定路径及其证明的性质。

Property 10. In covtree, there are infinite upward-going paths from the origin in which every node is a singleton.

性质 10。在协变树中，从原点出发存在无限向上路径，路径中每个节点都是单元素节点。

We call the subset of covtree which contains exactly all these paths singtree, since it is a tree of singletons. Figure 13 shows the first three levels of singtree.

我们将协变树中恰好包含所有这类路径的子集称为单元素树，因为它是由单元素节点构成的树。图 13 展示了单元素树的前三层结构。

To discuss singtree we will need the concept of the Newtonian order.

讨论单元素树需要用到牛顿序的概念。

Definition 23 (Newtonian causet/order). A Newtonian causet is a causet in which every element in level k is above every element in level $k-1$. A Newtonian order is an order whose representatives are Newtonian.

定义 23(牛顿因果集/牛顿序)。牛顿因果集是满足以下条件的因果集: k 层的每个元素都位于 $k-1$ 层的每个元素上方。牛顿序是所有表示都满足牛顿性的序。

In a Newtonian causet, every pair of elements which are unrelated have the same past and the same future, alluding to a notion of a Newtonian global time, hence its name. (Note however that a Newtonian order is not a good approximation of continuum Euclidean space.) A Newtonian causet is a ”stack of antichains,” and for any natural number N , the union of the first N levels is a past of a break. The local finiteness condition implies that every level whose elements are not maximal must be finite.

在牛顿因果集中，任意两个不可比的元素拥有相同的过去和相同的未来，这呼应了牛顿全局时间的概念，也因此得名。(但请注意，牛顿序并不是连续欧几里得空间的良好近似。) 牛顿因果集是“反链的堆叠”，对任意自然数 N ，前 N 层的并集就是一个断裂点的过去。局部有限性条件要求，所有包含非极大元的层都必须是有限的。

One can show that an order C is Newtonian if and only if for every natural number $n \leq |C|$ there is a unique n -order which is a stem in C . Thus, we have:

可以证明: 序 C 是牛顿序当且仅当对任意自然数 $n \leq |C|$ ，存在唯一的 n -序是 C 中的干。因此我们得到:

Property 11. A singleton $\{C_n\}$ is in singtree if and only if C_n is Newtonian.

性质 11。单元素节点 $\{C_n\}$ 属于单元素树当且仅当 C_n 是牛顿序。

Property 12. An infinite order C is Newtonian if and only if it is a certificate of a singtree path.

性质 12。无限序 C 是牛顿序当且仅当它是单元素树路径的证明。

If $\{C_n\}$ is a node in singtree, then it has exactly two direct descendants in singtree: $\left\{\widehat{C}_n\right\}$ and $\{D_{n+1}\}$, where D_{n+1} is the Newtonian order whose representative is constructed from a representative of C_n by adding a new element to its maximal level. If $\{C_n\}$ is a node in singtree, then it has exactly three direct descendants in covtree: its singtree descendants, $\left\{\widehat{C_n}\right\}$ and $\{D_{n+1}\}$, and the doublet $\left\{\widehat{C_n}, D_{n+1}\right\}$.

若 $\{C_n\}$ 是单元素树中的一个节点，则它在单元素树中恰好有两个直接后代: $\left\{\widehat{C}_n\right\}$ 和 $\{D_{n+1}\}$，其中 D_{n+1} 是牛顿序，它的表示由 C_n 的表示在其极大层添加一个新元素构造得到。若 $\{C_n\}$ 是单元素树中的一个节点，则它在协变树中恰好有三个直接后代: 它在单元素树中的两个后代 $\left\{\widehat{C_n}\right\}$ 和 $\{D_{n+1}\}$，以及双元素节点 $\left\{\widehat{C_n}, D_{n+1}\right\}$。

Given Property 12, it is now a simple matter to solve for the family of covtree dynamics in which the set of non-Newtonian orders is null: it is the set of covtree walks in which the walker stays in singtree with probability 1, i.e.,

根据性质 12，我们很容易就能求出非牛顿序集合为零集的协变动力学族: 这类动力学对应的协变树游走中，游走者以概率 1 停留在单元素树内，即

$$\mathbb{P}\left(\Gamma_n\right) = 0 \forall \Gamma_n \text{ not in singtree.} \tag{7}$$

This family of Newtonian dynamics acts as a proof of principle, illustrating how an understanding of covtree could allow one to solve for a dynamics with particular features. But, since these dynamics are unphysical, this is very much a case of ”looking under the lamp-post.” Where are we to look if not under the lamp-post? One avenue for exploration is to ask: what role, if any, do rogues play in the physics of covtree walks?

这类牛顿动力学可作为原理验证，说明对协变树的理解如何能帮助我们推导出具有特定特征的动力学。但由于这类动力学不具有物理性，这本质上还是“路灯下找钥匙”的做法。如果不在路灯下找，我们该去哪里寻找呢？一个探索方向是: 奇异路径在协变树游走的物理中究竟是否发挥作用?

Since in CSG models the set of rogues is null [13], identifying covtree dynamics which possess this property is a step towards understanding what form CSG dynamics take on covtree. Moreover, if following [13] we are to choose $\mathcal{R}$ to be our σ -algebra of observables then - unless the covtree measure on $\mathcal{R}(\mathcal{S})$ has a unique extension to $\mathcal{R}$ - one is faced with ambiguities both in interpretation and calculation. It is sufficient that the set of rogues be null for there to exist a unique extension, and therefore rogue-free dynamics are compatible with this approach.

由于在 CSG 模型中奇异路径集合为零集 [13]，识别出满足该性质的协变树动力学是理解 CSG 动力学在协变树上形式的重要一步。此外，若遵循文献 [13] 的思路，我们选择 $\mathcal{R}$ 作为可观测量的 σ -代数，那么除非 $\mathcal{R}(\mathcal{S})$ 上的协变树测度能唯一延拓到 $\mathcal{R}$，否则在解释和计算上都会存在歧义。只要奇异路径集合为零集就能保证存在唯一延拓，因此无奇异路径动力学与该方法相容。

One can draw an analogy between the condition that the set of rogues is null and the condition that the set of non-Newtonian orders is null: the former is the condition that the set of paths with more than one certificate is null, the latter the condition that the set of paths with more than one labeled certificate is null. However, while we were able to solve for the latter, solving for the former poses a new challenge because it is a limiting condition: at no finite stage of the covtree walk can the claim that the growing order is a rogue be verified or falsified. This is because for every node in covtree there exist both an infinite certificate which is a rogue and an infinite certificate which is not a rogue.

可以将异常集为空的条件与非牛顿阶集合为空的条件做类比: 前者对应拥有多个证书的路径集合为空，后者对应拥有多个标记证书的路径集合为空。但我们已经能求解后者，求解前者却带来了新挑战，因为它是一个极限条件: 在协变树游走的任意有限阶段，都无法证实或证伪「增长的阶是异常阶」这一论断。这是因为协变树的每个节点都同时存在属于异常阶的无限证书和不属于异常阶的无限证书。

This means that there is no rogue analogue to singtree. Instead, we must look for other ways to obtain rogue-free dynamics. We will see in section ”Covtree and Causal Set Cosmology” that pursuing the strictly stronger condition that the dynamics gives rise to infinitely many posts or breaks with unit probability is a promising route of particular interest for the causal set cosmology.

这意味着不存在对应单树的异常-free 类似结构。我们必须寻找其他方法来得到无异常动力学。我们将在「协变树与因果集合宇宙学」一节中看到，追求动力学以单位概率产生无穷多末端或分支的严格更强条件，是一条对因果集合宇宙学而言特别有意义的 promising 路径。

Self-Similarity

自相似性

One of covtree’s most interesting structural properties is its self-similarity. We now introduce this feature in advance of presenting its consequences for cosmic renormalization in section ”Covtree and Causal Set Cosmology”.

协变树最有趣的结构特性之一便是它的自相似性。我们现在先介绍这一特性，之后再在“协变树与因果集宇宙学”一节中阐述它对宇宙重整化的影响。

Recall that covtree is itself a causal set whose ground-set is Λ (Definitions 14 and 18).

回顾前文，协变树本身就是一个因果集，其底集为 Λ (定义 14 和 18)。

Definition 24 (Copy). A causal set Π contains a copy of some causal set Φ if there exists a convex subcauset $\Phi' \subseteq \Pi$ such that $\Phi \cong \Phi'$.

定义 24(拷贝): 若因果集 Π 中存在凸子因果集 $\Phi' \subseteq \Pi$ 满足 $\Phi \cong \Phi'$，则称 Π 包含某个因果集 Φ 的一个拷贝。

Definition 25 (Self-similar causal set). A causal set is self-similar if it contains infinitely many copies of itself.

定义 25(自相似因果集): 若一个因果集包含无穷多个自身的拷贝，则称它是自相似的。

For any finite order A , let $\Lambda_A \subset \Lambda$ be the convex subcauset of covtree which contains the node $\{\widehat{A}\}$ and everything above it.

对任意有限序 A，令 $\Lambda_A \subset \Lambda$ 为协变树中包含节点 $\{\widehat{A}\}$ 及其所有上方节点的凸子因果集。

Theorem 2. For any finite order A, Λ_A is a copy of covtree. Thus, covtree is self-similar.

定理 2: 对任意有限序 A, Λ_A， A, Λ_A 是协变树的一个拷贝。因此协变树是自相似的。

An illustration of covtree's self-similar structure is shown in Fig. 14.

协变树的自相似结构如图 14 所示。

The relationship between covtree and each of its copies Λ_A is given by the map $\mathcal{G}_A$.

协变树与其每一个拷贝 Λ_A 之间的关系由映射 $\mathcal{G}_A$ 给出。

Definition 26 (Break). A break in $\widetilde{C}$ is an ordered partition $\{\widetilde{A}, \widetilde{B}\}$ of $\widetilde{C}$ such that $a \prec b \forall a \in \widetilde{A}, b \in \widetilde{B}. \widetilde{A}$ and $\widetilde{B}$ are called the past and future of the break, respectively. An order C contains a break with past A if a representative of it contains a break with past $\widetilde{A}$, where $\widetilde{A}$ is some representative of A .

定义 26(中断): $\widetilde{C}$ 中的中断是 $\widetilde{C}$ 的有序划分 $\{\widetilde{A}, \widetilde{B}\}$，其中 $a \prec b \forall a \in \widetilde{A}, b \in \widetilde{B}. \widetilde{A}$ 和 $\widetilde{B}$ 分别称为中断的过去和未来。若序 C 的一个代表元包含中断，且该中断的过去为 $\widetilde{A}$ (其中 $\widetilde{A}$ 是 A 的某个代表元)，则称序 C 包含一个过去为 A 的中断。

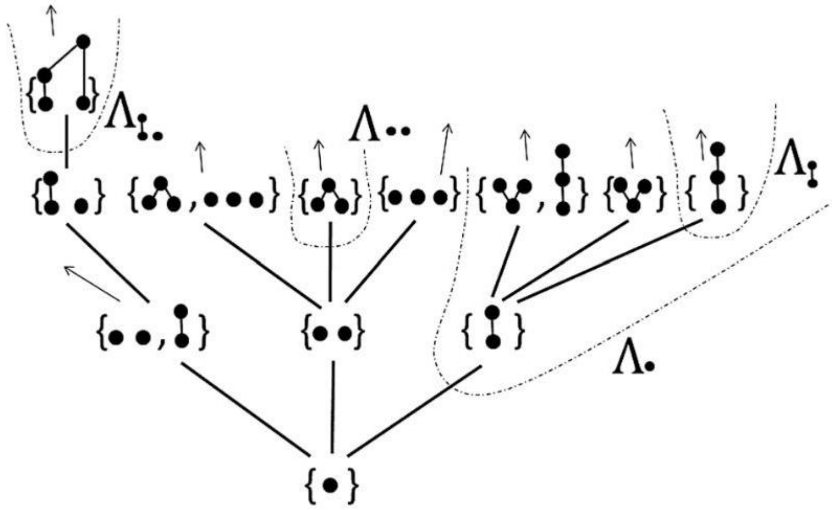


Fig. 14 The self-similar structure of covtree. The figure displays the first two levels of covtree in full and selected nodes from levels 3 and 4. The arrows indicate additional nodes not shown in the figure. The dashed lines indicate where a new copy of covtree begins. The ground-set Λ_A of each copy is indicated next to each dashed line. Figuratively, we can write $\Lambda = \Lambda_\varnothing$

图 14 协变树的自相似结构。本图完整展示了协变树的前两层，以及第 3 层和第 4 层的选中节点。箭头表示图中未画出的额外节点，虚线表示协变树新拷贝的起始位置。每个拷贝的底集 Λ_A 标注在对应虚线旁。形象地，我们可以记为 $\Lambda = \Lambda_\varnothing$

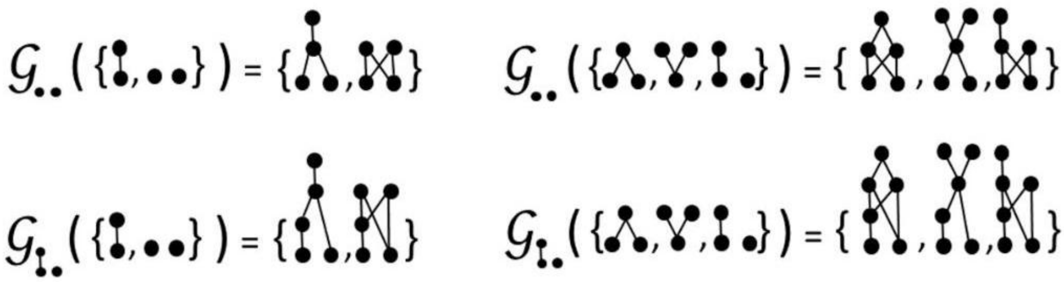


Fig. 15 Illustration of the operation $\mathcal{G}_A$

图 15 运算 $\mathcal{G}_A$ 的示例

An illustration is shown in Fig. 16a.

示例如图 16a 所示。

Definition 27 (The map $\mathcal{G}_A$). Given a finite order A and a set $\Gamma_n \in \Lambda$, the map $\mathcal{G}_A$ takes Γ_n to $\mathcal{G}_A(\Gamma_n)$, the set of orders which contain a break with past A and future $B_n \in \Gamma_n$, i.e.,

定义 27(映射 $\mathcal{G}_A$): 给定有限序 A 和集合 $\Gamma_n \in \Lambda$ ， 映射 $\mathcal{G}_A$ 将 Γ_n 映射为 $\mathcal{G}_A(\Gamma_n)$ ， 即所有包含过去为 A 、未来为 $B_n \in \Gamma_n$ 的中断的序的集合， 也就是

$\mathcal{G}_A(\Gamma_n) := \{C \mid C$ is an order containing a break with past A and future $B_n \in \Gamma_n\}$.

Examples are shown in Fig. 15.

实例如图 15 所示。

Covtree’s self-similarity can be stated as: for any finite order A, the map $\mathcal{G}_A : \Lambda \rightarrow \Lambda_A$ is an order-isomorphism. The maps $\mathcal{G}_A$ are order-preserving because they commute with the map $\mathcal{O}$ (Definition 17).

协变树的自相似性可以表述为: 对任意有限序 A，映射 $\mathcal{G}_A : \Lambda \rightarrow \Lambda_A$ 是一个序同构。映射 $\mathcal{G}_A$ 是保序的，因为它们与映射 $\mathcal{O}$ 可交换 (定义 17)。

Covtree and Causal Set Cosmology

Covtree 与因果集宇宙学

An attractive lens through which to study growth dynamics is that afforded by the cosmological paradigm of [24] which aims to explain the emergence of a flat, homogeneous, and isotropic cosmos directly from the quantum gravity era. (This paradigm pertains only to the causal set spacetime, not to any matter living on it. Whether a causal set is enough to give rise to matter degrees of freedom [9] or whether one requires additional structure such as a field living on the causal set is still unknown. Whichever the case may be, it is expected that this simplified cosmological paradigm will act as a guide to building a causal set cosmology.) In this heuristic model, the fundamental parameters of nature change their values as the universe goes through subsequent epochs of expansion and collapse, echoing evolutionary mechanisms proposed by J. A. Wheeler, L. Smolin and others to explain the values of the parameters of nature [42-45].

研究生长动力学的一个极具吸引力的视角，由文献 [24] 提出的宇宙学范式提供，该范式旨在直接从量子引力时代解释平坦、均匀且各向同性宇宙的起源。(该范式仅针对因果集时空，不涉及其上存在的任何物质。因果集是否足以产生物质自由度 [9]，还是需要额外结构如因果集上存在的场，目前仍不清楚。无论情况如何，该简化宇宙学范式都有望为构建因果集宇宙学提供指导。) 在这个启发式模型中，随着宇宙经历连续的膨胀与坍缩纪元，自然的基本参数会改变其取值，这呼应了 J.A. 惠勒、L. 斯莫林等人为解释自然参数取值提出的进化机制 [42-45]。

A causal set spacetime can be separated into epochs using the notions break (Definition 26) and post.

利用中断 (定义 26) 和 Post 的概念，可以将因果集时空划分为不同纪元。

Definition 28 (Post). $x \in \widetilde{C}$ is a post if it is related to all other elements in $\widetilde{C}$. The past (future) of a post $x \in \widetilde{C}$ is its non-inclusive past (future) in $\widetilde{C}$. An order C has a post with past A if a representative of it contains a post with past $\widetilde{A}$, where $\widetilde{A}$ is some representative of A.

定义 28(Post)。$x \in \widetilde{C}$ 是一个 Post，当且仅当它与 $\widetilde{C}$ 中的所有其他元素都存在关联。Post $x \in \widetilde{C}$ 的过去 (未来) 是它在 $\widetilde{C}$ 中的非包含过去 (未来)。序 C 存在一个带过去 A 的 Post，当且仅当它的一个代表元包含一个带过去 $\widetilde{A}$ 的 Post，其中 $\widetilde{A}$ 是 A 的某个代表元。

The notions of post and break are closely connected, since the following statements are equivalent: x is a post in $\widetilde{C}$; $\widetilde{C}$ admits a break $\{\widetilde{A}, \widetilde{B}\}$ where x is the unique maximal element of $\widetilde{A}$; and $\widetilde{C}$ admits a break $\{\widetilde{D}, \widetilde{E}\}$ where x is the unique minimal element of $\widetilde{E}$. An illustration is shown in Fig. 16b.

Post 和中断的概念联系紧密，以下表述等价: x 是 $\widetilde{C}$;$\widetilde{C}$ 中的 Post 等价于 $\widetilde{C}$;$\widetilde{C}$ 允许存在中断 $\{\widetilde{A}, \widetilde{B}\}$，其中 x 是 $\widetilde{A}$ 的唯一极大元；也等价于 $\widetilde{C}$ 允许存在中断 $\{\widetilde{D}, \widetilde{E}\}$，其中 x 是 $\widetilde{E}$ 的唯一极小元。图示见图 16b。

In a growth dynamics, the parameters of nature are the transition probabilities themselves or a set of couplings from which they can be computed. The mechanism by which these parameters change their value from epoch to epoch is called cosmic renormalization [25, 29]. The idea is that once the past of a break has been fully grown, the future of the break can be considered independently of the past as a growing causal set in its own right. The transition probabilities which govern the growth of the future are the ”renormalised” probabilities - a repackaging of the original transition probabilities together with information about the partial order structure of the past.

在生长动力学中，自然参数就是跃迁概率本身，或是可从中计算出跃迁概率的一组耦合。这些参数在不同纪元间改变取值的机制称为宇宙重整化 [25, 29]。其核心思想是，一旦中断的过去完全生长完成，中断的未来就可以独立于过去，作为一个自成一体的生长中因果集单独讨论。支配未来生长的跃迁概率就是“重整化后”的概率——它是原跃迁概率结合过去偏序结构信息重新封装得到的。

When the renormalization flow generated by successive epochs has certain features (for instance, that its stationary points grow causal sets with the desired cosmological features, that the basin of attraction of these stationary points is large, and that it contains an abundance of dynamics which are likely to give rise to posts/breaks.), the dynamics evolves into growing larger, flatter epochs as the universe cycles repeatedly. It is then only a matter of time until the universe displays the flat, homogeneous, and isotropic features we observe today.

当连续纪元产生的重整化流具有特定特征时 (例如，其不动点生长出的因果集符合预期宇宙学特征、这些不动点的吸引域很大, 且流中存在大量极可能产生 Post/中断的动力学), 随着宇宙反复循环, 动力学会逐渐演化出更大、更平坦的纪元。宇宙最终演化出我们如今观测到的平坦、均匀、各向同性特征只是时间问题。

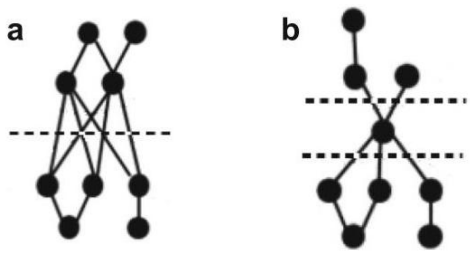


Fig. 16 Illustrations of a break and a post in a finite order. (a) An order with a single break. The suborder below the dotted line is the past of the break. (b) An order with two breaks and a post between them. The suborder below the lower dotted line is the past of the post

图 16 有限序中中断与 Post 的示例。(a) 仅含一个中断的序，虚线下方的子序是该中断的过去。(b) 含两个中断且二者之间存在一个 Post 的序，下方虚线下方的子序是该 Post 的过去

Certificates with Posts and Breaks

含柱与断的证书

The cosmological narrative above gives us a broad class of dynamics to aim for: dynamics which are likely to grow causal sets with a large number of posts or breaks. Which covtree dynamics fall into this category is an open question, but a first step in answering it has already been achieved through the classification of covtree paths whose certificates contain posts or breaks.

上述宇宙学叙事为我们指明了一大类可追寻的动力学: 这类动力学大概率生长出包含大量柱或断的因果集。哪类协树动力学属于这一范畴仍是一个开放问题，但要回答该问题，第一步已通过对证书含柱或断的协树路径分类完成。

Recall that $\widehat{A}$ denote the covering order of A (Definition 22), and let $\widehat{\widehat{A}}$ denote the covering order of $\widehat{A}$.

回顾: $\widehat{A}$ 表示 A 的覆盖序 (定义 22)，令 $\widehat{\widehat{A}}$ 表示 $\widehat{A}$ 的覆盖序。

Theorem 3. Let the order C be a certificate of the path $\mathcal{P}$.

定理 3. 设序 C 是路径 $\mathcal{P}$ 的一个证书。

1. C contains a break with past A if and only if $\{\widehat{A}\}$ is a node in $\mathcal{P}$.

1. 当且仅当 $\{\widehat{A}\}$ 是 $\mathcal{P}$ 中的一个节点时，C 包含一个过去为 A 的断。

2. C contains a post with past A if and only if $\{\widehat{\widehat{A}}\}$ is a node in $\mathcal{P}$.

2. 当且仅当 $\{\widehat{\widehat{A}}\}$ 是 $\mathcal{P}$ 中的一个节点时，C 包含一个过去为 A 的柱。

With Theorem 3 in hand, the challenge ahead is to write down a complete set of covtree transition probabilities which are likely to lead the random walker through a long sequence of nodes of the form $\{\widehat{A}\}$. Returning to our discussion of rogue-free dynamics (cf. section "Paths"), note that a rogue causet contains an infinite level [13] and as a result cannot contain an infinite sequence of posts or breaks. Therefore, requiring that the random walker passes through infinitely many nodes of the form $\{\widehat{A}\}$ is not only cosmologically relevant but also guarantees that the dynamics abhors rogue spacetimes.

有了定理 3，接下来的挑战是写出完整的协树转移概率集，使得随机游走者大概率会穿过一系列长的形式为 $\{\widehat{A}\}$ 的节点。回到我们对无反常动力学的讨论 (参见“路径”一节)，注意反常因果集包含无限层 [13]，因此无法包含无限长的柱或断序列。因此，要求随机游走者穿过无限多个形式为 $\{\widehat{A}\}$ 的节点不仅具有宇宙学相关性，还能保证动力学排斥反常时空。

Cosmic Renormalization on Covtree

共树的宇宙重整化

In addition to searching for dynamics which favor posts and breaks, constraints on the transition probabilities can be posed by requiring that the dynamics display certain behaviors under cosmic renormalization. To do so, we must first outline the explicit form that cosmic renormalization takes on covtree.

除了筛选倾向于生成柱和断的动力学之外，还可以通过要求动力学在宇宙重整化下展现特定行为来对跃迁概率施加约束。为此，我们首先需要梳理宇宙重整化在共树上的具体形式。

In the following, we denote a covtree transition probability by $\mathbb{P}(\Gamma_n \to \Gamma_{n+1})$. We use $\{\mathbb{P}\}$ to denote a complete set of covtree transition probabilities.

在下文中，我们用 $\mathbb{P}(\Gamma_n \to \Gamma_{n+1})$ 表示一个共树跃迁概率，用 $\{\mathbb{P}\}$ 表示完整的共树跃迁概率集。

Consider a growing order C which contains a break with past A . Theorem 3 tells us that we can consider the past A to have been fully grown when the random walker arrives at the node $\{\widehat{A}\}$. From this point onwards, we can consider the future as growing independently of this fixed past. We do so by acting on each node in Λ_A with $\mathcal{G}_A^{-1}$ (the inverse of Definition 27) since this effectively "deletes" the past A of the break. $\mathcal{G}_A^{-1}$ maps Λ_A to Λ (cf. Theorem 2) so that the growth of the future of the break (previously described by a walk on Λ_A) is now described as a walk on the whole of Λ and is governed by a new set of effective transition probabilities. Given a dynamics $\{\mathbb{P}\}$, the effective dynamics $\{\mathbb{P}_A\}$ which governs the growth of the future of a break with past A is given by

考虑一个增长序 C ，其中包含一个断，该断的过去为 A 。定理 3 告诉我们，当随机游走到达节点 $\{\widehat{A}\}$ 时，可以认为过去 A 已经完全生长完成。从这一点开始，我们可以认为未来的生长独立于这个固定的过去。我们通过对 Λ_A 中的每个节点作用 $\mathcal{G}_A^{-1}$ (定义 27 的逆操作) 来实现这一点，因为这一操作实际上“删除”了断的过去 A 。$\mathcal{G}_A^{-1}$ 将 Λ_A 映射到 Λ (参见定理 2)，因此断未来的生长 (此前描述为 Λ_A 上的游走) 现在可以描述为整个 Λ 上的游走，由一组新的有效跃迁概率控制。给定一个动力学 $\{\mathbb{P}\}$ ，控制带有过去 A 的断未来生长的有效动力学 $\{\mathbb{P}_A\}$ 可表示为

$$R_A : \{\mathbb{P}\} \mapsto \{\mathbb{P}_A\}, \mathbb{P}_A(\Gamma_n \to \Gamma_{n+1}) = \mathbb{P}(\mathcal{G}_A(\Gamma_n) \to \mathcal{G}_A(\Gamma_{n+1})). \tag{8}$$

Since the occurrence of a post with past A is equivalent to the occurrence of a break with past $\widehat{A}$, the effective dynamics which governs the growth of the future of a post is given by transformation $R_{\widehat{A}}$, obtained from transformation (8) via $A \to \widehat{A}$. An alternative formulation of renormalization after a post can be obtained by considering the post to be the minimal element of the future of a break (rather than the maximal element of the past of a break). In this case, the resulting effective dynamics is originary, i.e., $\mathbb{P}(\Gamma_1 \to \{1\}) = 1$, reflecting the condition that all elements must be related to the post. We denote the associated transformation by T_A (the apostrophe on the transition probabilities $\{\mathbb{P}'_A\}$ is used to distinguish between the images of $\{\mathbb{P}\}$ under R_A and T_A):

由于带有过去 A 的柱等价于带有过去 $\widehat{A}$ 的断，因此控制柱未来生长的有效动力学由变换 $R_{\widehat{A}}$ 给出，该变换可通过 $A \to \widehat{A}$ 从式 (8) 得到。柱后重整化还可以得到另一种表述: 将柱视为断未来的极小元 (而非断过去的极大元)。在这种情况下，得到的有效动力学是本源的，即 $\mathbb{P}(\Gamma_1 \to \{1\}) = 1$ ，反映了所有元素都必须与柱相关的条件。我们将对应的变换记为 T_A (跃迁概率 $\{\mathbb{P}'_A\}$ 上的撇号用于区分 $\{\mathbb{P}\}$ 在 R_A 和 T_A 下的像):

$$T_A : \{\mathbb{P}\} \to \{\mathbb{P}'_A\},$$

(9)

$$\mathbb{P}'_A(\Gamma_1 \to \{\}) = 1$$

$$\mathbb{P}'_A(\Gamma_n \to \Gamma_{n+1}) = \mathbb{P}(\mathcal{G}_A(\Gamma_n) \to \mathcal{G}_A(\Gamma_{n+1}))\, \forall \Gamma_n \succcurlyeq \{\}$$

$$\mathbb{P}'_A(\Gamma_n \to \Gamma_{n+1}) = 0 \text{ otherwise.}$$

Cosmic Renormalization as a Constraint

作为约束条件的宇宙重正化

The way in which the CSG models transform under cosmic renormalization is wellknown [25, 29]. In particular, it is known that the space of CSG models is closed under the cosmic renormalization transformations, that there exists a unique one-parameter family of stationary points, and that there are no higher-order cycles. Additionally, the effective CSG dynamics depends on the past A via two numbers only: the cardinality, a , and the number of maximal elements, r , of A . The remaining causal structure of A is forgotten, and the various renormalization transformations can be written in terms of powers of a single transformation, where the powers are simple functions of r and a . This is reminiscent of the form of the CSG transition probabilities which depend only on the cardinality and number of maximal elements of the past of the new-born element, a consequence of the Bell causality condition (cf. section ”The Structure of Covtree”).

CSG 模型在宇宙重正化下的变换方式是众所周知的 [25, 29]。尤其需要指出，已知 CSG 模型空间在宇宙重正化变换下是闭合的，存在唯一的单参数不动点族，且不存在高阶循环。此外，有效 CSG 动力学仅通过两个数依赖于过去 A : A 的基数 a 和极大元数量 r 。A 其余的因果结构都会被消去，各类重正化变换都可以用单个变换的幂次表示，幂次是 r 和 a 的简单函数。这和 CSG 跃迁概率的形式十分相似——新生成元的过去的跃迁概率仅依赖其基数和极大元数量，这是贝尔因果条件的推论 (参见章节“共形树的结构”)。

Requiring that the cosmic renormalization on covtree dynamics shares the features above could help us to better understand the form that the CSG models take on covtree. Additionally, new classes of physical dynamics could be obtained by requiring that covtree dynamics transform in particular ways. We summarize these ideas with some open questions: Is the condition on a covtree dynamics $\{\mathbb{P}\}$ that $\{\mathbb{P}_A\} = \{\mathbb{P}_B\}$ if and only if A and B have the same cardinality and number of maximal elements necessary for $\{\mathbb{P}\}$ to be a CSG dynamics? Is it sufficient? Does the factorization property of the CSG transformations bear any relation to the constraint on a covtree dynamics $\{\mathbb{P}\}$ that, for any finite order A , the renormalization transformation can be factorized as $R_A = R^{|A|}$ for some transformation R ? When such a factorization holds, the effective dynamics is independent of the causal structure of the past. Therefore, could the condition that R_A factorizes be interpreted as a causality condition on covtree dynamics?

如果要求共形树动力学下的宇宙重正化具备上述特征，能帮助我们更好地理解 CSG 模型在共形树上的形式。此外，通过要求共形树动力学按特定方式变换，我们可以得到新的物理动力学类。我们将这些思路总结为若干开放问题: 对于共形树动力学 $\{\mathbb{P}\}$ ，当且仅当 A 和 B 具有相同基数与极大元数量时满足 $\{\mathbb{P}_A\} = \{\mathbb{P}_B\}$ ，这个条件对于 $\{\mathbb{P}\}$ 成为 CSG 动力学是必要的吗？它是充分的吗？CSG 变换的分解性质和共形树动力学 $\{\mathbb{P}\}$ 的下述约束存在关联吗: 对任意有限阶 A ，重正化变换都可以分解为 $R_A = R^{|A|}$ ，其中 R 是某个变换？当满足该分解性质时，有效动力学独立于过去的因果结构。那么，R_A 可分解这个条件是否可以被解释为共形树动力学的一个因果性条件?

Variations

变体

The growth dynamics which we considered thus far, whether labeled or manifestly covariant, were constrained to grow those causal sets in which every element has a finite past. Thus, these dynamics can only describe cosmologies in which time has a beginning, and it is natural to ask whether it is possible to construct growth dynamics for cosmologies in which time has no beginning [20, 34]. From the outset, conceptual problems arise. Perhaps the most pressing of these is that, within the framework of labeled sequential growth, growing an infinite past requires that elements be born to the past of existing ones, making it (nearly if not entirely) impossible to conceive of the growth process as a physical phenomenon. However, one can identify a set of physically meaningful observables - namely, the convex-events which describe the convex suborders contained in the growing causal set - and this sets the stage for adapting covtree for two-way infinite growth. The variations presented here first appeared in [34].

截至目前我们讨论的增长动力学，无论是带标记还是显式协变形式，都受限于增长所有元素的过去都是有限的因果集。因此，这些动力学只能描述时间有开端的宇宙学，自然会引出一个问题: 能否为时间无开端的宇宙学构建增长动力学 [20, 34]。这类问题从一开始就存在概念层面的难题，其中最紧迫的问题或许是: 在带标记序贯增长框架下，增长出无限过去要求新元素诞生于已有元素的过去，这使得几乎 (甚至完全) 无法将增长过程视为一种物理现象。不过，我们仍然可以找到一组具有物理意义的可观测量——即描述增长因果集中包含的凸子序的凸事件——这为将协变树适配为双向无限增长模型铺垫了基础。此处介绍的变体最早发表于文献 [34]。

Terminology for Two-Way Infinite Causal Sets

双向无限因果集术语

A causal set is past-finite (future-finite) if every element is preceded (succeeded) by at most finitely many others. A causal set is past-infinite (future-infinite) if it is not past-finite (future-finite). A causal set is two-way infinite if it is both past-infinite and future-infinite.

若因果集的每个元素至多有有限个前驱 (后继)，则称该因果集是过去有限 (未来有限) 的。若因果集不是过去有限 (未来有限) 的，则称其为过去无限 (未来无限) 的。若因果集同时满足过去无限和未来无限，则称其为双向无限的。

In the previous sections, the sample space of our growth process was $\widetilde{\Omega}$, the set of labeled causal sets on the ground-set $\mathbb{N}$. Every infinite past-finite causal set is order-isomorphic to some $\widetilde{C} \in \widetilde{\Omega}$, or equivalently we can say that every countably infinite past-finite causal set has a natural labeling by $\mathbb{N}$. The converse is also true: only past-finite causal sets can have a natural labeling by $\mathbb{N}$.

在前文中，我们生长过程的样本空间是 $\widetilde{\Omega}$ ，即基集 $\mathbb{N}$ 上带标记因果集的集合。每个无限过去有限因果集都序同构于某个 $\widetilde{C} \in \widetilde{\Omega}$ ，等价地说，每个可数无限过去有限因果集都存在由 $\mathbb{N}$ 给出的自然标记。反之也成立: 只有过去有限的因果集可以拥有由 $\mathbb{N}$ 给出的自然标记。

To describe two-way infinite causal sets, we must extend our index set from $\mathbb{N}$ to $\mathbb{Z}$. Every two-way infinite causal set has a labeling by $\mathbb{Z}$, though the converse is not true since past-finite causal sets with infinitely many minimal elements and future-finite causal sets with infinitely many maximal elements also admit labelings by $\mathbb{Z}$ [46, 47] .

为描述双向无限因果集，我们必须将索引集从 $\mathbb{N}$ 扩展到 $\mathbb{Z}$ 。每个双向无限因果集都存在由 $\mathbb{Z}$ 给出的标记，但反之不成立: 具有无穷多极小元的过去有限因果集，以及具有无穷多极大元的未来有限因果集，也可以拥有由 $\mathbb{Z}$ [46, 47] 给出的标记。

Fig. 17 The ”infinite comb” (left) and the disjoint union of the infinite comb with a single element (right) are convex-rogues since they contain the same convex suborders as each other

图 17 “无限梳”(左) 和无限梳与单个元素的不交并 (右) 都是凸异常序，因为二者包含完全相同的凸子序

We generalize the definition of a labeled causet (Definition 1) to include those causal sets whose ground-set is an interval of integers (including the infinite intervals $\mathbb{Z}$ and $\mathbb{N}$) and whose partial order is compatible with the order on $\mathbb{Z}$ (i.e., $x \prec y \Rightarrow x < y$). From here onwards, orders are defined to be order-equivalence classes of this extended class of labeled causal sets.

我们将带标记因果集的定义 (定义 1) 推广，使其包含基集为整数区间 (包括无限区间 $\mathbb{Z}$ 和 $\mathbb{N}$)、且偏序与 $\mathbb{Z}$ 上的序兼容 (即符合 $x \prec y \Rightarrow x < y$) 的因果集。此后，我们将序定义为这个扩展类中带标记因果集的序等价类。

Let C and D denote orders with representatives $\widetilde{C}$ and $\widetilde{D}$, respectively. We will say that C is a convex suborder in D if $\widetilde{D}$ contains a copy of $\widetilde{C}$. In that case we may also say that C is a convex suborder in $\widetilde{D}$. If additionally C is an n -order, we say that C is an n -suborder in D or in $\widetilde{D}$. We will say that C is a convex-rogue if there exists another order $D \ncong C$ which has the same n -suborders as C for all n . In that case we say that C and D are a convex-rogue pair. We may also refer to $\widetilde{C}$ and $\widetilde{D}$ as convex-rogues or as a convex-rogue pair.

设 C 和 D 分别表示以 $\widetilde{C}$ 和 $\widetilde{D}$ 为代表元的序。若 $\widetilde{D}$ 包含一份 $\widetilde{C}$ 的拷贝，我们称 C 是 D 的凸子序，此时也可以称 C 是 $\widetilde{D}$ 的凸子序。若额外满足 C 是一个 n -序，我们称 C 是 D 或 $\widetilde{D}$ 的一个 n -子序。若存在另一个序 $D \ncong C$ ，使得对所有 n ，它和 C 拥有完全相同的 n -子序，则称 C 是凸异常序，此时称 C 和 D 为凸异常对，也可以将 $\widetilde{C}$ 和 $\widetilde{D}$ 称为凸异常序或凸异常对。

An example of a convex-rogue pair is shown in Fig. 17.

图 17 给出了一个凸异常对的示例。

Convex-Covtree

凸协变树

The first variation of covtree which we will encounter is convex-covtree, whose definition is obtained from the definition of covtree by relacing n -stem with n - suborder. Thus, $\Gamma_n \subset \Omega(n)$ is a node in convex-covtree if and only if there exists some order C whose set of n -suborders is Γ_n . We call C the convex-certificates of Γ_n . The ordering of the nodes in convex-covtree is as follows: for $m < n, \Gamma_m \prec \Gamma_n$ if Γ_m is the set of m -suborders of the elements in Γ_n . One way to think about the ordering in convex-covtree is to pick an n -order in Γ_n and delete a maximal or minimal element of it to form an $(n-1)$ -order. Then Γ_n is directly above the node Γ_{n-1} that contains all $(n-1)$ -orders which can be formed in this way.

我们介绍的第一种协变树变体就是凸协变树，它的定义是将协变树定义中的 n 干替换为 n 子序得到的。因此，$\Gamma_n \subset \Omega(n)$ 是凸协变树中的节点当且仅当存在某个序 C ，其所有 n 子序构成的集合恰好是 Γ_n 。我们将 C 称为 Γ_n 的凸证书。凸协变树中节点的排序规则如下: 对 $m < n, \Gamma_m \prec \Gamma_n$ ，若 Γ_m 是 Γ_n 中元素的所有 m 子序构成的集合，则……我们可以这样理解凸协变树的排序关系: 从 Γ_n 中取出一个 n 序，删除它的一个极大元或极小元，得到一个 $(n-1)$ 序；那么 Γ_n 就直接位于节点 Γ_{n-1} 的上方，节点 Γ_{n-1} 包含所有能通过这种方式得到的 $(n-1)$ 序。

The nodes in the first three levels of convex-covtree are shown in Fig. 18.

凸协变树前三层的节点如图 18 所示。

Convex-covtree bears some similarities to covtree. In particular, every inex-tendible path in convex-covtree has a convex-certificates, allowing us to interpret a random walk on convex-covtree as a covariant process of growth: the growing order is a convex-certificates of the path which is traced by the random walk. Each node in the path corresponds to a covariant property of the growing order, i.e., Γ_n is the set of n -suborders of the growing order.

凸协变树与标准协变树存在诸多相似之处: 具体来说，凸协变树中每条不可延拓路径都有一个凸证书，这让我们可以将凸协变树上的随机游走解释为一个协变增长过程: 增长中的序就是随机游走所遍历路径的凸证书。路径上的每个节点都对应增长序的一个协变性质，即 Γ_n 是增长序的所有 n 子序构成的集合。

But unlike covtree, convex-covtree contains maximal nodes so that some of its inextendible paths are finite. This is a consequence of the fact that the existence of a finite-convex certificate does not guarantee the existence of an infinite one. In particular it is known that, if C_n is not the n -chain or the n -antichain, the cardinality of the convex-certificates of $\{C_n\}$ is bounded from above by n^2 . Finite inextendible paths are exactly the paths that contain such a singleton $\{C_n\}$. This does not mean that every such singleton is a maximal node, although the maximal nodes are always singletons containing their own - and their path's - unique convex-certificates. Thus, every finite inextendible path has a certificate. The converse is not true, some finite orders are certificates of no path at all.

但和协变树不同的是，凸协变树存在极大节点，因此它的部分不可延拓路径是有限的。这一性质源于: 有限凸证书的存在不能保证无限凸证书的存在。已知的结论是，若 C_n 不是 n 链或 n 反链，则 $\{C_n\}$ 的凸证书的基数以 n^2 为上界。有限不可延拓路径恰好是包含这类单元素 $\{C_n\}$ 的路径。这并不意味着所有这类单元素集合都是极大节点，但极大节点永远是单元素集合，其中仅包含自身 (同时也是对应路径) 的唯一凸证书。因此，所有有限不可延拓路径都存在凸证书，反之不成立——部分有限序根本不是任何路径的证书。

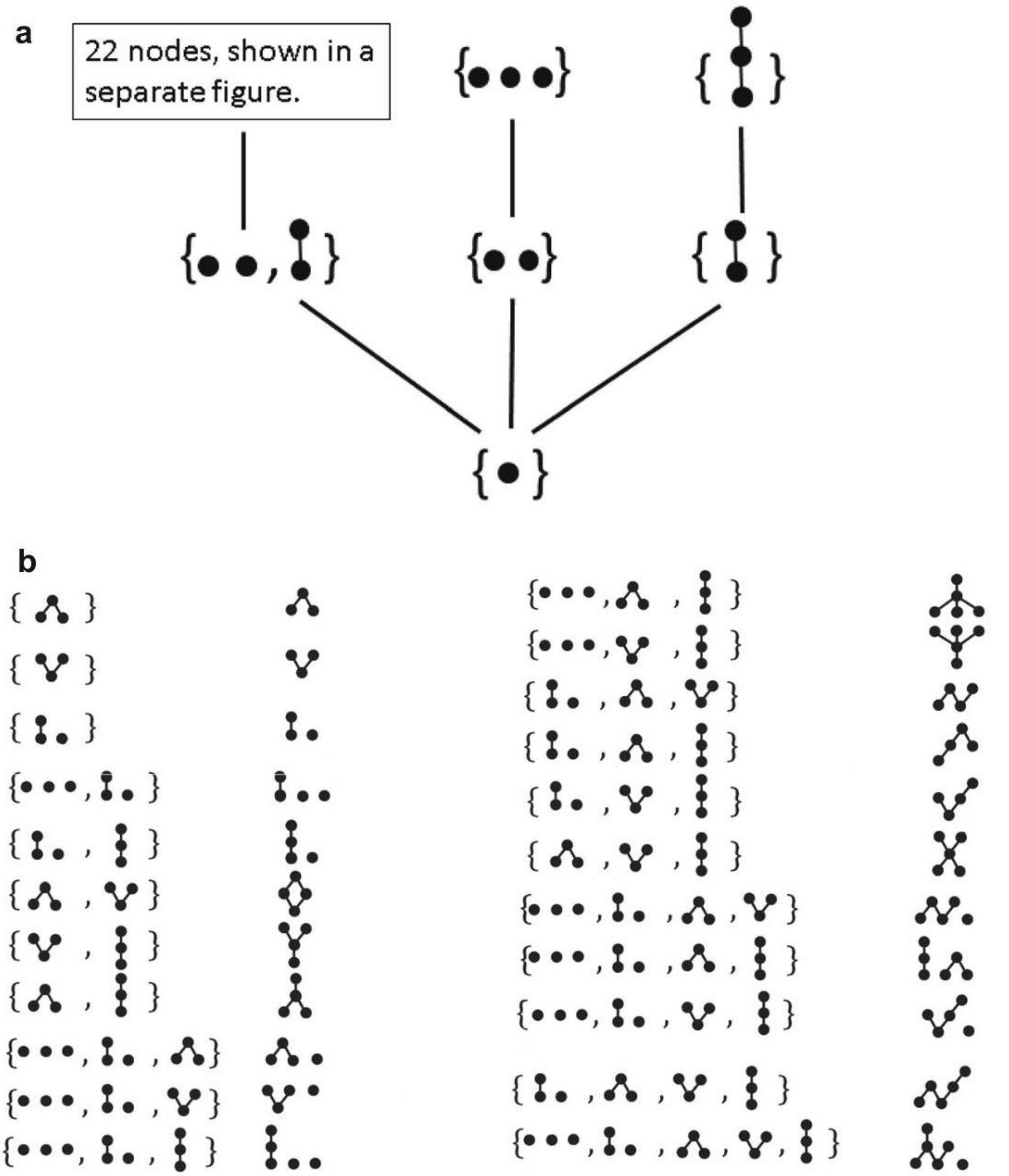


Fig. 18 The first three levels of convex-covtree. (a) The first three levels of convex-covtree. (b) 22 nodes of convex-covtree and their convex-certificates. These are the level 3 nodes which appear directly above the doublet

图 18 凸协变树的前三层。(a) 凸协变树的前三层；(b) 凸协变树的 22 个节点及其凸证书，这些都是直接位于二元组上方的第三层节点

Turning our attention to infinite paths, we note that every infinite order is a convex-certificates of some infinite inextendible path, and conversely, every infinite path in convex-covtree has a convex-certificates (the proof is similar to that of Theorem 1). A path has more than one convex-certificates if its convex-certificates are convex-rogues and, in this case, which convex-certificates is the growing order is up for interpretation (e.g., we can consider all convex-certificates of a given path to be physically equivalent).

> 讨论无限路径时我们发现: 每个无限序都是某条无限不可延拓路径的凸证书；反之，凸协变树中的每条无限路径也都存在凸证书 (证明与定理 1 的证明类似)。若路径的凸证书是凸反常序，则该路径会拥有多个凸证书；这种情况下，到底哪个凸证书对应增长中的序可以有不同解释 (例如，我们可以认为给定路径的所有凸证书都是物理等价的)。

$\mathbb{Z}$ -Covtree

$\mathbb{Z}$ -协方差树

A second variation of covtree is $\mathbb{Z}$ -covtree, defined as a truncation of convex-covtree: $\mathbb{Z}$ -covtree is the subtree of convex-covtree which contains exactly all nodes that have a convex-certificates (with a representative) naturally labeled by $\mathbb{Z}$. Like covtree, $\mathbb{Z}$ -covtree contains no maximal elements, and every inextendible path has at least one convex-certificates. Moreover, one can show that every inextendible path has at least one convex-certificates whose representative is naturally labeled by $\mathbb{Z}$, allowing us to consider the set of such orders as the sample space.

> 协方差树的第二种变体是 $\mathbb{Z}$ -协方差树，定义为凸协方差树的截断: $\mathbb{Z}$ -协方差树是凸协方差树的子树，恰好包含所有带有由 $\mathbb{Z}$ 自然标记凸证书 (带代表元) 的节点。和普通协方差树一样，$\mathbb{Z}$ -协方差树不包含极大元，且每条不可延拓路径都至少有一个凸证书。此外可以证明，每条不可延拓路径都至少存在一个凸证书，其代表元由 $\mathbb{Z}$ 自然标记，因此我们可以将这类序的集合视为样本空间。

Like in covtree, the σ -algebra of observables is generated by the certificate sets associated with the nodes. For each Γ_n in $\mathbb{Z}$ -covtree, let $\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)$ denote the set of labeled convex-certificates of Γ_n whose ground-set is $\mathbb{Z}$. A dynamics is given by a measure μ on the σ -algebra generated by the $\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)$ ’ s, where $\mu(\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)) = \mathbb{P}(\Gamma_n)$.

> 和普通协方差树一样，可观测量的 σ 代数由与节点关联的证书集合生成。对于 $\mathbb{Z}$ -协方差树中的每个 Γ_n ， 令 $\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)$ 表示 Γ_n 中底集为 $\mathbb{Z}$ 的带标记凸证书的集合。动力学由测度 μ 给出，定义在由所有 $\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)$ 生成的 σ 代数上，其中 $\mu(\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)) = \mathbb{P}(\Gamma_n)$ 。

We saw in section ”The Sample Space, Algebra, and Measure” that covtree’s observable algebra is equivalent to the stem algebra, $\mathcal{R}(\mathcal{S})$, generated by the stem sets of Equation (3). We can pursue the analogy between stems and convex sets further by defining for each finite order C_n the set $\mathrm{convex}(C_n)$ to be the collection of labeled causets with ground-set $\mathbb{Z}$ which contain C_n as a convex suborder. A convex-event is any set which can be generated from the $\mathrm{convex}(C_n)$ ’ s via countable set operations (i.e., a convex-event is an element of the σ -algebra generated by the $\mathrm{convex}(C_n)$ ’ s). Each convex-event is a covariant measurable event with a clear physical meaning - it corresponds to a logical combination of statements about which finite orders are convex suborders in the growing causet. One can show that the σ -algebra generated by the $\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)$ is equal to the σ -algebra generated by the $\mathrm{convex}(C_n)$.

我们在“样本空间、代数与测度”一节中已经看到，协方差树的可观测量代数等价于由式 (3) 的干集合生成的干代数 $\mathcal{R}(\mathcal{S})$。我们可以进一步拓展干与凸集之间的类比: 对每个有限序 C_n，定义集合 $\text{convex}(C_n)$ 为所有底集为 $\mathbb{Z}$、且将 C_n 包含为凸子序的带标记因果集的全体。凸事件是可通过对凸 (C_n) 做可数集合运算生成的任意集合 (即凸事件是由凸 (C_n) 生成的 σ 代数中的元素)。每个凸事件都是一个具有明确物理意义的协变可测事件——它对应关于“哪些有限序是增长因果集中的凸子序”这一类命题的逻辑组合。可以证明，由 $\text{cert}_{\mathbb{Z}}(\Gamma_n)$ 生成的 σ 代数与由 $\text{convex}(C_n)$ 生成的 σ 代数相等。

The upshot is that $\mathbb{Z}$ -covtree furnishes a growth framework for two-way infinite causal sets, with the caveat that past-finite causal sets with infinitely many minimal elements and future-finite causal sets with infinitely many maximal elements must be suppressed by the dynamics.

结论是，$\mathbb{Z}$ -协方差树为双向无限因果集提供了一个增长框架，但需要注意: 具有无限多个极小元的过去有限因果集，以及具有无限多个极大元的未来有限因果集，必须被动力学排除。

$\mathbb{N}$ -Covtree

$\mathbb{N}$ -共树形

The success of $\mathbb{Z}$ -covtree in providing a growth framework for two-way infinite causal sets based on the premise that the observables are convex-events raises the question: is it possible to define growth dynamics for past-finite causal sets in which the observables are convex-events? One can try doing so by defining a third variation of covtree, namely: the subtree of convex-covtree which contains exactly all nodes that have a convex-certificates (with a representative) naturally labeled by $\mathbb{N}$. We call this variation $\mathbb{N}$ -covtree.

$\mathbb{Z}$ -共树形基于可观测物是凸事件这一前提，成功为双向无限因果集提供了增长框架，由此引出一个问题: 是否能为过去有限的因果集定义以凸事件作为可观测物的增长动力学？我们可以尝试定义共树形的第三种变体: 凸共树形中恰好包含所有节点的子树，这些节点都拥有自然由 $\mathbb{N}$ 标记的凸证明 (带代表元)，我们将该变体称为 $\mathbb{N}$ -共树形。

While the definition of $\mathbb{N}$ -covtree is completely analogous to that of $\mathbb{Z}$ -covtree, the resulting structure is not. In particular, there are inextendible paths $\mathcal{P}$ in $\mathbb{N}$ - covtree which do not have a convex-certificates labeled by $\mathbb{N}$. By our definition of $\mathbb{N}$ -covtree, every node in $\mathcal{P}$ has a convex-certificates labeled by $\mathbb{N}$ - but there may be no such convex-certificates common to all nodes in $\mathcal{P}$.

$\mathbb{N}$ -共树形的定义和 $\mathbb{Z}$ -共树形的定义完全类似，但二者得到的结构并不相同。具体而言，在 $\mathbb{N}$ -共树形中存在不可延拓路径 $\mathcal{P}$，这些路径没有由 $\mathbb{N}$ 标记的凸证明。根据我们对 $\mathbb{N}$ -共树形的定义，$\mathcal{P}$ 中的每个节点都拥有一个由 $\mathbb{N}$ 标记的凸证明，但可能不存在一个同时属于 $\mathcal{P}$ 中所有节点的这类凸证明。

This means that convex-events cannot act as observables for past-finite causal sets since there is no surjection from the set of infinite past-finite orders to the set of N-covtree paths and the measure space construction we described in section ”The Sample Space, Algebra, and Measure” doesn’t carry through.

这意味着凸事件无法作为过去有限因果集的可观测物，因为无限过去有限序的集合到 N-共树形路径的集合不存在满射，我们在“样本空间、σ 代数与测度”一节中描述的测度空间构造也无法成立。

One can understand this stark difference between $\mathbb{Z}$ -covtree and $\mathbb{N}$ -covtree using the language of metric spaces. For any two orders C and D , let $C \sim D$ if and only if C and D are a convex-rogue pair, i.e., if they share the same n -suborders for all n . Let $\Omega_{\mathbb{N}}$ and $\Omega_{\mathbb{Z}}$ denote the sets of orders which have a representative with ground-set $\mathbb{N}$ and $\mathbb{Z}$, respectively. Let $\Omega_{\mathbb{N}}/\sim$ and $\Omega_{\mathbb{Z}}/\sim$ be quotient spaces under the convex-rogue equivalence relation, so that their elements are equivalence classes of orders denoted by $[C]$, etc. We can consider these quotient spaces as metric spaces with metric $d([C],[D]) = \frac{1}{2^n}$, where n is the largest integer for which representatives of $[C]$ and $[D]$ have the same sets of n -suborders. Given a node Γ_n in convex-covtree, we can associate with it a subset $[\mathrm{cert}_{\mathbb{N}}(\Gamma_n)] \subseteq \Omega_{\mathbb{N}}/\sim$, namely, the set of elements of $\Omega_{\mathbb{N}}/\sim$ whose representatives are convex-certificates of Γ_n and similarly $[\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)] \subseteq \Omega_{\mathbb{Z}}/\sim$. Given a convex-covtree path $\mathcal{P} = \Gamma_1 \prec \Gamma_2 \prec \ldots$, we can associate with it the sets $[\mathrm{cert}_{\mathbb{N}}(\mathcal{P})] = \bigcap_{\Gamma_n \in \mathcal{P}} [\mathrm{cert}_{\mathbb{N}}(\Gamma_n)]$ and $[\mathrm{cert}_{\mathbb{Z}}(\mathcal{P})] = \bigcap_{\Gamma_n \in \mathcal{P}} [\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)]$. The metric space $(\Omega_{\mathbb{Z}}/\sim, d)$ is complete, and therefore by Cantor’s lemma $[\mathrm{cert}_{\mathbb{Z}}(\mathcal{P})]$ is non-empty whenever all the $[\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)]$ is non-empty for all $\Gamma_n \in \mathcal{P}$. On the other hand, the metric space $(\Omega_{\mathbb{N}}/\sim, d)$ is not complete, and therefore $[\mathrm{cert}_{\mathbb{N}}(\mathcal{P})]$ can be empty even when $[\mathrm{cert}_{\mathbb{N}}(\Gamma_n)]$ is non-empty for all $\Gamma_n \in \mathcal{P}$.

我们可以用度量空间的语言来理解 $\mathbb{Z}$ 协树与 $\mathbb{N}$ 协树之间的显著差异。对任意两个序关系 C 和 D ，当且仅当 C 和 D 构成凸 rogue 对 (即它们对所有 n 都有相同的 n 子序) 时，记为 $C \sim D$ 。分别设 $\Omega_{\mathbb{N}}$ 和 $\Omega_{\mathbb{Z}}$ 是基集为 $\mathbb{N}$ 和 $\mathbb{Z}$ 的序的代表集合。设 $\Omega_{\mathbb{N}}/\sim$ 和 $\Omega_{\mathbb{Z}}/\sim$ 是凸 rogue 等价关系下的商空间，其元素为 $[C]$ 等序等价类。我们可以将这些商空间视为带有度量 $d([C],[D]) = \frac{1}{2^n}$ 的度量空间，其中 n 是满足 $[C]$ 和 $[D]$ 的代表拥有相同 n 子序集合的最大整数。给定凸协树中的一个节点 Γ_n ，我们可以为它关联一个子集 $[\mathrm{cert}_{\mathbb{N}}(\Gamma_n)] \subseteq \Omega_{\mathbb{N}}/\sim$ ，即 $\Omega_{\mathbb{N}}/\sim$ 中代表为 Γ_n 凸证书的元素集合，同理可得 $[\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)] \subseteq \Omega_{\mathbb{Z}}/\sim$。给定一条凸协树路径 $\mathcal{P} = \Gamma_1 \prec \Gamma_2 \prec \ldots$ ，我们可以为它关联集合 $[\mathrm{cert}_{\mathbb{N}}(\mathcal{P})] = \bigcap_{\Gamma_n \in \mathcal{P}} [\mathrm{cert}_{\mathbb{N}}(\Gamma_n)]$ 和 $[\mathrm{cert}_{\mathbb{Z}}(\mathcal{P})] = \bigcap_{\Gamma_n \in \mathcal{P}} [\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)]$。度量空间 $(\Omega_{\mathbb{Z}}/\sim, d)$ 是完备的，因此根据康托尔引理，只要对所有 $\Gamma_n \in \mathcal{P}$ 对应的 $[\mathrm{cert}_{\mathbb{Z}}(\Gamma_n)]$ 均非空，那么 $[\mathrm{cert}_{\mathbb{Z}}(\mathcal{P})]$ 就非空。另一方面，度量空间 $(\Omega_{\mathbb{N}}/\sim, d)$ 不是完备的，因此即使对所有 $\Gamma_n \in \mathcal{P}$ 对应的 $[\mathrm{cert}_{\mathbb{N}}(\Gamma_n)]$ 均非空，$[\mathrm{cert}_{\mathbb{N}}(\mathcal{P})]$ 仍可以是空集。

For example, consider the path,

例如，考虑如下路径,

$$\mathcal{P} = \{.\} < \{1, ..\} < \{1, \Lambda\vee\} < \{1, \Lambda\vee\} < \ldots \tag{10}$$

Each node $\Gamma_n \in \mathcal{P}$ has a convex-certificates $D^n \in \Omega_{\mathbb{N}}$, as illustrated in Fig. 19. These convex-certificates (technically, the equivalence classes in $\Omega_{\mathbb{N}}/\sim$ of which they are representatives) form a Cauchy sequence in $(\Omega_{\mathbb{N}}/\sim, d)$, where $d\left([D^n],[D^{n-1}]\right) = \frac{1}{2^n}$. The limit of the sequence is the order D shown in Fig. 19. Since D is two-way infinite, we know that $[D] \notin \Omega_{\mathbb{N}}/\sim$ so that $(\Omega_{\mathbb{N}}/\sim, d)$ is not a complete metric space. Additionally, D is the only certificate of $\mathcal{P}$, so $\mathcal{P}$ is an example of a path in $\mathbb{N}$ -covtree which has no past-finite convex-certificates.

每个节点 $\Gamma_n \in \mathcal{P}$ 都对应一个凸证书 $D^n \in \Omega_{\mathbb{N}}$，如图 19 所示。这些凸证书 (严格来说，它们是 $\Omega_{\mathbb{N}}/\sim$ 中等价类的代表元) 在 $(\Omega_{\mathbb{N}}/\sim, d)$ 中构成柯西序列，其中 $d\left([D^n],[D^{n-1}]\right)=\frac{1}{2^n}$。该序列的极限就是图 19 所示的序 D。由于 D 是双向无穷的，可知 $[D] \notin \Omega_{\mathbb{N}}/\sim$，因此 $(\Omega_{\mathbb{N}}/\sim, d)$ 不是完备度量空间。此外，D 是 $\mathcal{P}$ 唯一的凸证书，因此 $\mathcal{P}$ 是 $\mathbb{N}$ -协树中不存在过去有限凸证书的路径的一个例子。

Finally, note that the machinery of metric spaces can be used to give an alternative proof to Theorem 1 which stated that every inextendible path in covtree has a certificate. In this case, the metric space is $(\Omega_{\mathbb{N}}/{\sim_R}, \delta)$ where $\sim_R$ is the rogue equivalence relation (cf. Definition 10) and $\delta\left([C]_R,[D]_R\right)=\frac{1}{2^n}$ where n is the largest integer for which representatives of $[C]_R$ and $[D]_R$ have the same sets of n -stems.

最后需要注意的是，可以用度量空间的方法为定理 1 给出另一种证明，该定理指出协树中每条不可延拓路径都存在证书。在本情形中，度量空间为 $(\Omega_{\mathbb{N}}/{\sim_R}, \delta)$，其中 $\sim_R$ 是反常等价关系 (参见定义 10)，且度量为 $\delta\left([C]_R,[D]_R\right)=\frac{1}{2^n}$，其中 n 是使得 $[C]_R$ 和 $[D]_R$ 的代表元拥有相同 n -干集的最大整数。

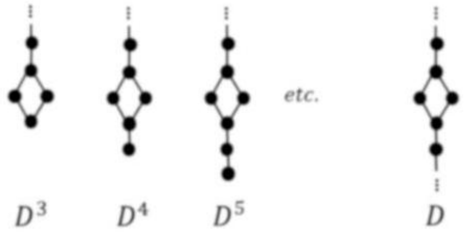


Fig. 19 The order $D \in \Omega_{\mathbb{Z}}$ shown on the right is a convex-certificates of the path $\mathcal{P}$. Every node in $\mathcal{P}$ has a convex-certificates in $\Omega_{\mathbb{N}}$: D^3 is a convex-certificates of $\Gamma_n \in \mathcal{P}$ only for $n \leq 3, D^4$ is a convex-certificates of $\Gamma_n \in \mathcal{P}$ only for $n \leq 4, D^5$ is a convex-certificates of $\Gamma_n \in \mathcal{P}$ only for $n \leq 5$, etc. There is no order in $\Omega_{\mathbb{N}}$ which is a convex-certificates of every node in $\mathcal{P}$

图 19 右侧所示的序 $D \in \Omega_{\mathbb{Z}}$ 是路径 $\mathcal{P}$ 的凸证书。$\mathcal{P}$ 中的每个节点都在 $\Omega_{\mathbb{N}}$: D^3 中拥有凸证书，$D \in \Omega_{\mathbb{Z}}$ 仅为 $\Gamma_n \in \mathcal{P}$ 中 $n \leq 3, D^4$ 的凸证书，$n \leq 3, D^4$ 仅为 $\Gamma_n \in \mathcal{P}$ 中 $n \leq 4, D^5$ 的凸证书，$n \leq 4, D^5$ 仅为 $\Gamma_n \in \mathcal{P}$ 中 $n \leq 5$ 的凸证书，以此类推。在 $\Omega_{\mathbb{N}}$ 中不存在同时是 $\mathcal{P}$ 中每个节点的凸证书的序。

Discussion

讨论

Covtree and its variations form a manifestly covariant, label-independent framework through which growth dynamics for causal sets can be defined. Their study is motivated by the need to understand general covariance within quantum gravity, and one approach to doing so is to ask whether one can formulate the laws of physics in a way which makes reference only to physical (and not to gauge) degrees of freedom. Modern theoretical physics has thus far favored gauge theories, but covtree is proof that at least within the discrete setting of causal set theory, it is possible to do away with gauge degrees of freedom. In future, the unified nature of quantum gravity may also offer new possibilities in this direction. A second motivation for the development of these covariant dynamics has been that the labeled sequential growth dynamics have thus far resisted quantization and there is hope that a covariant formulation may offer a new route to quantum dynamics. Indeed, a label-independent formulation may prove necessary since concepts unrelated to each other in our current theories, such as general covariance and quantum interference, may prove inseparable in a full theory of quantum gravity.

> 协变树及其变体构成了一个显然协变、与标签无关的框架，可通过该框架定义因果集的生长动力学。研究它们的动机来自理解量子引力中广义协变性的需求，实现这一点的一种思路是探究能否以仅参考物理 (而非规范) 自由度的方式表述物理定律。现代理论物理学迄今一直倾向于规范理论，但协变树证明，至少在因果集理论的离散背景下，完全摒弃规范自由度是可行的。未来，量子引力的统一性或许也会为这个方向带来新的可能。开发这类协变动力学的第二个动机是，带标签的顺序生长动力学迄今一直无法量子化，人们希望协变表述能为量子动力学提供一条新路径。事实上，不依赖标签的表述很可能是必要的，因为在完整的量子引力理论中，当前理论中互不相关的概念 (比如广义协变性和量子干涉) 或许最终是不可分割的。

One of the interesting issues which are highlighted by covtree is the interplay between the notions of ”local” and ”global.” One can consider a causal set as a local object and an order as its global counterpart, since in a causal set one can identify individual elements and in an order one cannot. Similarly, in the labeled sequential growth, it is known exactly which element is born at each stage of the growth, but in a covtree growth, such an element cannot be identified in general. On the other hand, there is also a certain flavor of locality in covtree since at each finite stage of the covtree process we know which stems are contained in the growing causal set, but we don’t know how they fit together (there is no God’s eye view, only the viewpoint of somewhat local observers). In a similar vein, one can ask whether the failure of the convex-events to form a set of observables for past-finite causal sets can be interpreted as a statement about the local/global nature of observables: the event that the growing causal set contains some n -suborder pertains to the whole of the causal set, but the statement that it contains some n -stem is anchored to the antichain of minimal elements.

> 协变树凸显的一个有趣问题是“局域”概念和“全局”概念之间的相互作用。我们可以将因果集视为局域对象，将序视为其全局对应物，因为在因果集中可以识别单个元素，而在序中无法做到。类似地，在带标签的顺序生长中，生长的每个阶段确切知道哪一个元素诞生，但在协变树生长中，一般无法识别这样的元素。另一方面，协变树中也存在某种局域性: 在协变树过程的每个有限阶段，我们都知道生长中的因果集包含哪些干，但不知道它们如何拼接 (不存在上帝视角，只有某种局域观测者的视角)。同理，我们可以追问: 对于过去有限的因果集，凸事件无法构成可观测量集，这一点能否解读为关于可观测量局域/全局性质的表述: 生长中的因果集包含某个 n -序的事件关乎整个因果集，但包含某个 n -干的表述锚定在极小元的反链上。

The condition that the causal set contains a break or a post is a global condition, since it pertains to every element in the causal set. As a result, the occurrence of a break or a post with a given past can be falsified but never verified at a finite stage of the (inherently local) sequential growth dynamics, forcing our hand to perform post-selection in order to discuss cosmic renormalization. This post-selection is no longer necessary in the covtree framework, since the occurrence of a break or a post with a given past is synonymous with the random walker passing through a particular node of the form $\{\widehat{A}\}$. We find that the global occurrence of a break or a post manages to give us a glimpse of locality in the covtree process since it is exactly when the walker passes through one of these nodes that one can discern which causal set has been grown thus far - $\widehat{A}$ - and identify a new-born element - the maximal element of $\widehat{A}$. It is then that one can most convincingly associate a notion of growth with the covtree random walk.

因果集包含断点或后置元的条件是一个全局条件，因为它关乎因果集中的每一个元素。因此，在本质上局域的顺序生长动力学的有限阶段，只能证伪、永远无法证实给定过去的断点或后置元存在，这使得我们不得不进行后选择来讨论宇宙重整化。这种后选择在协变树框架中不再必要，因为给定过去的断点或后置元存在等价于随机游走经过 $\{\widehat{A}\}$ 形式的特定节点。我们发现，断点或后置元的全局存在得以让我们在协变树过程中一窥局域性，因为正是当游走经过这类节点时，我们才能分辨迄今生长出的因果集——$\widehat{A}$ ——并识别出新生元素，即 $\widehat{A}$ 的极大元。只有这时，我们才能最令人信服地将生长的概念和协变树随机游走联系起来。

This forms yet another motivation - in addition to those provided by the causal set cosmological paradigm and by the search rogue-free dynamics - to seek covtree dynamics which give rise to an infinite sequence of breaks or posts.

这除了因果集宇宙学范式和寻找无 rogue 动力学给出的动机之外，为寻找能产生无穷序列断点或后置元的协变树动力学提供了又一个动机。

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